





Stable Performance Under Sensor Failure of Local Positioning Systems

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Abstract. Local Positioning Systems are an active topic of research in the field of autonomous navigation. Its application in difficult complex scenarios has meant a solution to provide stability and accuracy for high-demanded applications. In this paper, we propose a methodology to enhance Local Positioning Systems performance in sensor failure contexts. This fact guarantees system availability in adverse conditions. For this purpose, we apply a Genetic Algorithm Optimization in a five-sensor 3D TDOA architecture in order to optimize the sensor deployment in nominal and adverse operating conditions. We look for a trade-off between accuracy and algorithm convergence in the position determination in four (failure conditions) and five sensor distributions. Results show that the optimization with failure consideration outperforms the non-failure optimization in a 47% in accuracy and triples the convergence radius size in failure conditions, with a penalty of only 6% in accuracy during normal performance.

Keywords: CRLB · Genetic Algorithms · Localization · LPS · Sensor failure

1 Introduction

Autonomous navigation has meant a challenge for scientific development over the last few years. The high accuracy required has entailed the interest in Local Positioning Systems (LPS) where the positioning signal paths are reduced between targets and architecture sensors. This fact reduces noise and uncertainties through the minimization of the global architecture errors with respect to Global Navigation Satellite Systems (GNSS).

LPS cover a defined and known space with architecture sensors where the capabilities of the system are maximized. LPS properties depend on the measurement of the physical

magnitude used for the determination of the target location: time [1], power [2], frequency [3], angle [4], phase [5] or combinations of them [6].

Among these systems, the most extended are time-based models due to their reliability, stability, robustness and easy-to-implement hardware architectures. Time-based positioning computes the total or relative travel time of the positioning signals from the target to the receivers generating two different system conceptions: total time-of-flight-Time of Arrival (TOA) [7] - and relative time-of-flight -Time Difference of Arrival (TDOA) [8] systems-.

TDOA systems compute the relative time between the reception of the positioning signal in two different architecture sensors. Therefore, the synchronization of these systems is optional. Asynchronous TDOA architectures measures time differences in a single clock of a coordinator sensor [9] while in synchronous TDOA all architecture sensors must be synchronized to compute all together the time measurements.

Time relative measurements lead to hyperboloid surfaces of possible location of targets. Every two architecture sensors a hyperboloid equation is obtained while only $(n - 1)$ independent equations are achieved from n different sensors [10]. The required number of sensors to determine unequivocally the target location is 5 sensors for 3-D positioning in these methodologies. However, the intersection of three different hyperboloids in TDOA systems leads to two different potential solutions. Nevertheless, these solutions are not able to be discarded from a mathematical point of view.

In one of our previous works [11], we have shown that a reliable unique solution to the intersection of three hyperboloids or spheres can be obtained through the maximization of the distance between the two potential solutions by means of Genetic Algorithms (GA). We achieve this result by applying Taylor-based algorithms [12] from an initial iteration point which must be close enough to the final solution. Node deployment showed to have a direct impact for this achievement.

The sensor distribution has also relation with the accuracy of the LPS. Cramér-Rao Lower Bound (CRLB) [13, 14] derivations allow the characterization of the White Gaussian Noise (WGN) in the time measurements, estimating the minimum achievable error in positioning systems [15]. This has allowed us to study the node deployment optimization in TDOA systems by means of GA [16, 17]. The reason of the use of heuristic techniques relies on the NP-Hard problem solution of the 3D sensor deployment in LPS and it is widespread in the literature [18–20].

However, the consideration of sensor failures has not yet been considered for LPS sensor distribution optimizations. In this paper, we propose for this purpose a GA optimization for the 3D node deployment in a TDOA system with five architecture sensors that can suffer from sensor failures. We perform a multi-objective optimization in which we look for a trade-off between the accuracy of the system with five sensors and every combination of four nodes in a defined environment of an LPS. This methodology will ensure the availability of the system with acceptable accuracy in case of sensor failures in the architecture nodes.

The remainder of the paper is organized as follows: the algorithm for the target unequivocal location determination is introduced in Sect. 1, the CRLB modeling is presented in Sect. 2, the GA and the fitness function are detailed in Sect. 3 and Sects. 4 and 5 show the results and conclusions of the present paper.

2 Taylor-Based Positioning Algorithm in TDOA Systems

Relative time measurements in TDOA systems lead to hyperboloid equations of possible target locations. Numerical methods are needed to solve the intersection of these non-linear equations. Taylor-Based algorithms in TDOA systems are linearizations of the equation of the time difference of arrival:

$$\begin{aligned}
 R_{ij} &= d_{ij} = d_{Ei} - d_{Ej} = ct_{ij} = c(t_i - t_j) \\
 &= \sqrt{(x_E - x_i)^2 + (y_E - y_i)^2 + (z_E - z_i)^2} \\
 &\quad - \sqrt{(x_E - x_j)^2 + (y_E - y_j)^2 + (z_E - z_j)^2}
 \end{aligned} \tag{1}$$

where R_{ij} and d_{ij} represent the distance difference of the signal travel from the emitter to sensors i and j , d_{Ei} and d_{Ej} are total distance from the emitter (E) to sensors i and j , c is the speed of the radioelectric waves, t_{ij} is the time difference of arrival measured in the architecture sensors, t_i and t_j is the total time of flight of the positioning signal from emitter to receivers i and j respectively and (x_E, y_E, z_E) , (x_i, y_i, z_i) and (x_j, y_j, z_j) are the Cartesian coordinates of the target and the sensors i and j .

Taylor approximation truncated on first order is applied in Eq. 1 to linearize the equation from an initial iteration point (x_0, y_0, z_0) :

$$R_{ij} = ct_{ij} = R_{ij_0} + \frac{\partial R_{ij}}{\partial x} \Delta x + \frac{\partial R_{ij}}{\partial y} \Delta y + \frac{\partial R_{ij}}{\partial z} \Delta z \tag{2}$$

where R_{ij_0} is the range difference of arrival in the initial iteration point, $\frac{\partial R_{ij}}{\partial x}$, $\frac{\partial R_{ij}}{\partial y}$ and $\frac{\partial R_{ij}}{\partial z}$ are the partial derivatives of the range differences measured in the i and j architecture sensors particularized in the initial iteration point. The application of Eq. 2 to every pair of sensors of the TDOA architecture leads to the following relation, that enables the obtainment of the target location.

$$\Delta \mathbf{P} = (\mathbf{H}^t \mathbf{H})^{-1} \mathbf{H}^t \Delta \mathbf{R} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} \tag{3}$$

where \mathbf{H} is the partial derivative matrix, and $\Delta \mathbf{P}$ represents the incremental values from the last iteration point in each space direction which supposes the unknown of the equation.

3 CRLB Modeling in TDOA Systems

CRLB is an unbiased estimator of the lowest variance of a parameter. Its usage in the localization field is widespread [21–25] since it allows us to determine the minimum achievable error by the system. The uncertainties introduced in the measurements depend on the distance traveled by the positioning signal from the emitter to the architecture

sensors in a heteroscedastic noise consideration. Recent studies [14] developed a matrix form of the CRLB considering heteroscedasticity in time measurements:

$$FIM_{mn} = \left[\frac{\partial h(TS)}{\partial x_m} \right]^T R^{-1}(TS) \left[\frac{\partial h(TS)}{\partial x_n} \right] + \frac{1}{2} tr \left\{ R^{-1}(TS) \left[\frac{\partial R(TS)}{\partial x_m} \right] R^{-1}(TS) \left[\frac{\partial R(TS)}{\partial x_n} \right] \right\} \quad (4)$$

where **FIM** indicates the Fisher Information Matrix, m and n are the sub-indexes of the estimated parameters in the **FIM**, TS is the target sensor Cartesian coordinates, $\mathbf{h}(TS)$ is a vector that contains the travel of the signal in the TDOA architecture to compute a time measurement:

$$h_{TDOA_i} = \|TS - AS_i\| - \|TS - AS_j\| \quad i = 1, \dots, N_{AS} \quad j = 1, \dots, N_{AS} \quad (5)$$

being AS_i and AS_j the coordinates of the architecture sensors i and j and N_{AS} the number of sensors involved in the position determination. $\mathbf{R}(TS)$ is the covariance matrix of the time measurements in the architecture sensors. The covariance matrix is built with a heteroscedastic noise consideration in the sensors modeled by a Log-normal path loss propagation model [17] obtaining the following variances:

$$\sigma_{TDOA_{ij}}^2 = \frac{c^2}{B^2(P_t/P_n)} PL(d_0) \left[\left(\frac{d_{Ei}}{d_0} \right)^n + \left(\frac{d_{Ej}}{d_0} \right)^n \right] \quad i = 1, \dots, N_{AS} \quad j = 1, \dots, N_{AS} \quad \text{where } i \neq j \quad (6)$$

where B is the signal bandwidth, P_T is the transmission power, P_n the mean noise level determined through the Johnson-Nyquist equation, n the path loss exponent, d_0 the reference distance from which the path loss propagation model is applied and $PL(d_0)$ the path-loss in the reference distance.

The trace of the inverse of the Fisher Information Matrix (**J**) provides the uncertainties associated with each variable to estimate, i.e. the three Cartesian coordinates of the target for a 3D positioning. The location accuracy is directly evaluated through the Root Mean Squared Error (RMSE), which is computed based on the trace of the **J** matrix.

4 GA Optimization

The strong influence of the sensor distribution in the LPS performance enables the maximization of their capabilities through the optimization of their sensor placement. This approach is especially critical in complex 3D environments, where the most important source of positioning error is promoted by the sensor distribution.

In our previous works [17], a GA for optimizing sensor distributions in 3D irregular environments is presented. The proposed methodology allows a modular definition of the optimization region and the reference surface for locating the sensors of the positioning architecture. In addition, the procedure allows the election of different selection

techniques, percentage of elitism, crossover methodologies, mutation types, and convergence criteria. After the choice of the optimization method, the next step is the definition of the fitness function. In this case, we look for a multi-objective optimization for the combined maximization of the TDOA system accuracy in 3D environments when the whole architecture is available and when only four sensors are accessible, limited by the size of the convergence regions that allow the correct execution of the Taylor-based positioning algorithm. The fulfillment of these objectives guarantees the robustness of the TDOA architectures in adverse conditions of operation. The fitness function is detailed hereafter:

$$\begin{aligned}
 ff = & \sum_1^{Comb} \left\{ \frac{C_1}{NT} \sum \left\{ 1 - \frac{\left[\left(\frac{1}{RMSE_{ref}} \right) - \left(\frac{1}{\mathbf{RMSE}_{4sensors}} \right) \right]^2}{\left(\frac{1}{RMSE_{ref}} \right)^2} \right\} \right. \\
 & + \frac{C_2}{NT} \sum \left\{ \frac{\left[\left(\frac{1}{Dist_{ref}} \right) - \left(\frac{1}{\mathbf{Dist}} \right) \right]^2}{\left(\frac{1}{D_{ref}} \right)^2} \right\} \\
 & \left. + C_3 \sum \frac{\left\{ 1 - \frac{\left[\left(\frac{1}{RMSE_{ref}} \right) - \left(\frac{1}{\mathbf{RMSE}_{Ncs}} \right) \right]^2}{\left(\frac{1}{RMSE_{ref}} \right)^2} \right\}}{NT} - C_4 \frac{\sum_{i=1}^{NCS} BL_i}{NCS} \right\} \quad (7)
 \end{aligned}$$

where $Comb$ is the number of groups of four sensors which are obtainable based on the total number of architecture sensors, NT is the number of analyzed points, $RMSE_{ref}$ is the reference accuracy, $\mathbf{RMSE}_{4sensors}$ is the vector that contains the CRLB evaluation for each point at analysis with each combination of 4 sensors, $Dist_{ref}$ indicates the reference distance for the convergence criteria, \mathbf{Dist} represents the vector that specifies the convergence evaluation in terms of the distance between the two possible solutions for each point at study, \mathbf{RMSE}_{Ncs} is the vector that contains the CRLB analysis for each point at study when all architecture sensors are available, C_k are coefficients for calibration of the individual summands of the fitness function and BL_i is the penalization factor associated with the existence of sensors in banned regions (if they exist).

5 Results

In this section, the results of the optimization for sensor failure in TDOA architectures are detailed. Initially, a 3D complex scenario has been designed for carrying out the optimization, proving the adaptability of the proposed methodology in any environment.

In Fig. 1, the term TLE represents the Target Location Environment which defines the region where the targets are possibly located. For this simulation, the TLE region extends from 0.5 to 5 m of elevation from the base surface, based on a division of 10 m in x and y Cartesian coordinates, and 1 ms in z coordinate. This ensures the correct evaluation and continuity of the accuracy and convergence analysis.

The NLE area expresses the Node Location Environment, which indicates all possible sensor locations. In the case of the NLE region, the height of the sensors is limited in the

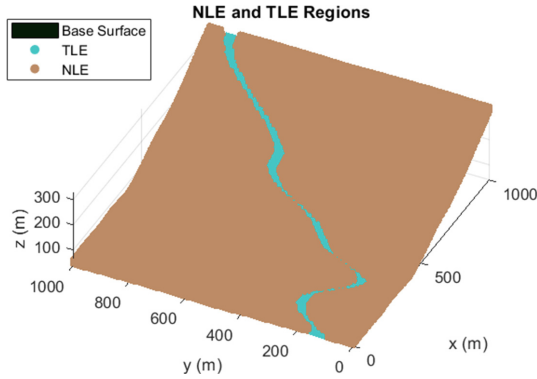


Fig. 1. The scenario of simulations. The reference surface is depicted in grey tones. NLE and TLE regions are respectively shown in orange and purple colors.

range 3 to 20 m from the base surface. The discretization of the NLE region depends on the codification of the individuals of the GA, precisely on the longitude of the chromosomes implemented. In this optimization, the resolution of the NLE area varies in the three Cartesian coordinates from 0.5 to 1 m, alluring a fine setting in the optimization of each sensor. Tables 1 and 2 show the principal parameters of configuration for the positioning system and the GA applied for the optimization.

Table 1. Parameters of configuration for the positioning system operation [15, 25, 26].

Parameter	Value
Transmission power	100 W
Mean noise power	-94 dBm
Frequency of emission	1090 MHz
Bandwidth	80 MHz
Path loss exponent	2.16
Antennae gains	Unity
Time-Frequency product	1

Values presented in Table 1 have been chosen in an attempt to stand for a generic positioning technology, expressed by the typical parameters of transmission power, frequency of emission and bandwidth. The GA configuration is based on the following aspects: population of 120 individuals with binary codification, Tournament 3 as selection procedure with 2% of elitism, single-point crossover, single-point mutation with a probability of 5%, and 90% of equal individuals as convergence criteria. This election allows the trade-off between fitness function maximization and processing time. For more information about the genetic operators and the design of the GA [17]. In addition,

C_k coefficients are defined as unity, searching for a solution with normal condition predominance in the final sensor deployment, but with good failure conditions performance. This GA was coded in the MATLAB software following every of these considerations. The results after the optimization process are shown for distributions of 5 sensors. The results for the optimized sensor placement with failure consideration, 5 sensors nominal operating conditions and convergence maximization (Conf. 1) are provided in Figs. 2 and 3 when two of the sensors are not available.

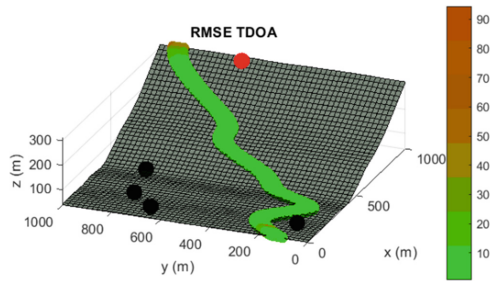


Fig. 2. Accuracy analysis in terms of CRLB for the optimized distribution of 5 sensors under possible failure of two arbitrary sensors of the architecture. Black spheres indicate active sensors and the red sphere symbolizes the failing sensor.

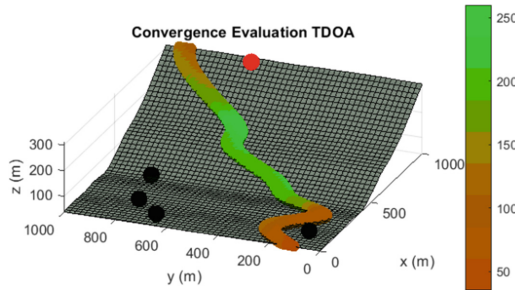


Fig. 3. Convergence radius analysis for the optimized distribution of 5 sensors under possible failure of two arbitrary sensors of the architecture.

In Table 2, a comparison between the optimized sensor distribution for sensor failure (Conf. 1) and the optimized sensor placement of 5 sensors without malfunction consideration and convergence maximization (Conf. 2) is supplied. It should be stressed that this last optimization is carried out through a fitness function with the direct evaluation of the CRLB for 5 sensors and the last term of the Eq. 7.

Results of Table 2 reveal that the optimization carried out in Conf. I not only minimizes the CRLB when only 4 sensors are accessible but also maximizes the region where the Taylor-based positioning algorithm is able to operate. The beauty of this combined multi-objective optimization is that the accuracy of the four-sensor combinations in failure conditions is increased by 47% while the accuracy of the normal operating

Table 2. Comparison between two optimized sensor distributions: with (Conf. 1) and without (Conf. 2) failure consideration.

	Operating condition	RMSE (m)		Convergence radius (m)	
		Mean	Max	Min	Mean
Conf. 1	Non-failure	1.154	3.234	40	171.687
	Failure	7.061	181.325	32	167.312
Conf. 2	Non-failure	1.085	7.114	4	145.807
	Failure	13.293	>300	0	37.841

five sensor distribution (Conf. 1) is reduced less than a 6% with regards to conventional node deployments (Conf. 2) that only consider the five-sensor optimization.

This new optimization procedure considering sensor failures does guarantee the robustness of the positioning system in complex conditions of operations, and the design of architectures considering these situations.

6 Conclusions

In this paper, a method to guarantee the system accuracy under sensor failure is proposed. We address the possible sensor malfunctioning or ineffective link between target and architecture sensors which are key factors in LPS actual deployments.

For this purpose, we have defined a 3D scenario in which a five-sensor distribution of a TDOA architecture is deployed in order to achieve practical results. The possible failure of two sensors in adverse operating conditions leads to the solution of the ambiguity in the target position determination with four receivers. We have proved that an unequivocal solution can be attained through the use of Taylor-Based positioning algorithms in combination with an optimized node location looking for a maximization of the distance between the two possible solutions in the four-sensor TDOA problem.

Accuracy analysis must be also carried out in both nominal and failure operating conditions. Therefore, we perform a multi-objective optimization of the node location by means of a Genetic Algorithm. This optimization looks for the maximization of the convergence of the positioning algorithms and the accuracy of the architecture to solve this NP-Hard problem.

Results show that both accuracy and convergence can be achieved under every possible sensor failure condition. The optimization considering only four effective links with the architecture sensors in failure conditions triples the values of the convergence region and increases the accuracy in 47% regarding to conventional optimizations that do not consider these adverse situations.

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