Pecvnia, núms. 16/17 (enero-diciembre 2013), pp. 41-54

APPLICATION OF A COHERENT RISK MEASURE IN THE PRICE CALCULATION OF AN INCOME INSURANCE (ANNUITIES)

Monserrat Hernández Solís[1]
montserrath@cee.uned.es

Faculty of Economics at UNED

Emma Berenguer Cárceles
ebercar@upo.es

Faculty of Economics at Pablo de Olavide University

fecha de recepción: 01/09/2013
fecha de aceptación: 06/02/2014

Resumen

Una práctica común que realizan las entidades aseguradoras es la de modificar las tasas de mortalidad instantánea al aplicar el principio de prima neta con el fin de hacer frente a las desviaciones desfavorables de la siniestralidad. Este documento proporciona una respuesta matemática a esta cuestión mediante la aplicación de la función de distorsión de potencia de Wang. Tanto la prima neta y la función de distorsión de Wang son medidas de riesgo coherentes, siendo este último aplicado por primera vez en el campo delos seguros de vida.
Utilizando las leyes de Gompertz y Makeham primero calculamos la prima a nivel general y en una segunda parte, se aplica el principio de cálculo de la prima basado en función de distorsión de potencia de Wang para calcular el recargo sobre la prima de riesgo ajustada. El precio de prima única de riesgo se ha aplicado a una forma de cobertura de seguro de supervivencia, el seguro de rentas.
La principal conclusión que puede extraerse es que mediante el uso de la función de distorsión, la nueva tasa instantánea de mortalidad es directamente proporcional a un múltiplo, que es justamente el exponente de esta función y hace que el riesgo de longevidad sea mayor. Esta es la razón por la prima de riesgo ajustada es superior a la prima neta.

---

[1] Montserrat Hernández, Department of Company Finance and Accounting in the Faculty of Economics at UNED, Paseo Senda del Rey nº 11, Madrid-28040; Emma Berenguer, Department of Financial Economics and Accounting in the Faculty of Economics at Pablo de Olavide University, Carretera de Utrera Km.1, Sevilla-41013.

Palabras clave: Seguro de rentas; Axiomas coherentes; Medida coherente del riesgo; Función de distorsión; Recargo implícito; Leyes de supervivencia.

Abstract

Modification of instantaneous mortality rates when applying the net premium principle in order to cope with unfavorable deviations in claims, is common practice carried out by insurance companies. This paper provides a mathematical answer to this matter by applying Wang's power distortion function. Both net premium and Wang's distortion function are coherent risk measures, the latter being first applied to the field of life insurance.
Using the Gompertz and Makeham laws we first calculate the premium at a general level and in a second part, the principle of premium calculation based on Wang´s power distortion function is applied to calculate the adjusted risk premium surcharge. The risk single premium pricing has been applied to a form of survival insurance coverage called Annuities.
The main conclusion to be drawn is that by using the distortion function, the new instantaneous mortality rate is directly proportional to a multiple, which happens to be the exponent of this function and causes longevity risk to be greater. This is why the adjusted risk premium is higher than the net premium.

Keywords: Annuities; Coherent axioms; Coherent risk measure; Distortion function; Implicitly surcharged; Survival laws.

1. Location of study

Life insurance is a transaction, the duration of which exceeds one year (Bowers, Newton, Gerber and Jones, 1997). It consists of single or multiple benefits and single or multiple considerations. The benefit represents the capital guaranteed by the company at the time when the loss covered by the policy occurs, which is paid out to the insured party or beneficiary, depending on the type of insurance, while the consideration represents the premium to be paid by the policyholder to qualify for coverage of the risk borne. Insurance with survival coverage has been selected in this article, the so-called income insurance, and its application to two laws of survival, the Gompertz and Makeham laws. In this method, a negative claims scenario for the company means that the insured parties have survived longer than expected (greater longevity). The net premium principle is the coherent risk measure which is currently used to adjust life insurance rates in the industry. The problem posed is that it does not provide an adjusted premium, either express or implied. This is the reason insurance companies add a safety margin as standard practice, as a percentage, to the probabilities of survival p_x, or use a mortality table which has probabilities of death that are lower than the considered human group. This can be interpreted as a decrease of both the instantaneous with a multiple. Thus, The intended objective is to avoid adverse deviations of the actual claim with respect to the expected claim. It is shown that the modification of the instantaneous amount of mortality is reflected in the fact that this amount is multiplied by the ratio $\frac{1}{\rho}$, the parameter ρ having the interpretation of participant risk aversion (Tse, 2009). This article shows an alternative principle to the net premium principle, the principle based on the Wang distortion function. This author already proposed an adjusted premium calculation principle for non-life insurance, based on the coherent risk measure (Artzner and Delbaen, 1999), distorted life expectancy with the Wang distortion function in potential form (Instantaneous proportional conversion of the amount), being the form of the distortion function $g(u) = u^{\frac{1}{\rho}}$, with the condition that the parameter $\rho \geq 1$ in order for it to be a coherent risk measure.

This paper continues the line of research begun by Wang but applied to the field of life insurance, proposing a premium calculation principle, the distorted life expectancy with the proportional converted distortion function of the instantaneous amount (Hernández et al., 2013). It is demonstrated that setting rates by using this premium calculation principle produces the same effect of decreasing the instantaneous amount than by using a mortality table with lower probabilities of death for this type of selected insurance. This is why this premium calculation principle may be considered an alternative to the net premium principle.

2. Risks and risk measures versus premium calculation principles

For an insurer to optimally manage the company, it is necessary to analyze the risks that affect insurers within the field of life insurance, since the analysis includes the measurement or quantification, as well as obtaining good protection against these risks and trying to prevent them.

Risk management, geared towards life insurance, is considered to be the perfect coordination between insurable risk (the death or survival of the annuitant) and an appropriate reduction of insurance costs. This kind of risk management is a goal all companies should reach for, because as the theorem of Modigliani-Miller (1958) states, efficient risk management can lead to a number of positive effects, such as a reduction of taxes, due to a reduction in the variability of cash-flow; being beneficial to a company, in the sense that they can have better access to capital markets than individual investors; Producing an increase in the value of the company in case of bankruptcy (in addition to making it less likely in terms of occurrence), as well as facilitating the acquisition of optimal investments.

The main risks that insurance companies face are various.

- Market risk: It occurs for both life insurance and nonlife insurance. It is also known as investment risk, due to the fact that insurers invest funds that in principle are allocated for the payment of benefits arising from claims filings covered by the policy, but funds that will not be paid out until after many years have gone by (given that the primary coverage is the death of the insured party). Logically, these funds must be invested in highly liquid assets for whenever a loss might occur, triggering the pay-out of guaranteed benefits under the policy. It is a risk arising from uncertainty regarding the returns on investments made by the insurer. The companies will invest the collected premiums and which are still not intended for the efficient coverage of claims. A compromise will be required between performance and the risk borne.

- Liquidity risk: This is the lack of opportunity to monetize the assets in which the insurer has invested its resources, and therefore the inability of meeting the payment of benefits of the life insurance beneficiary upon the death of the insured party.
- Credit Risk: This is the risk that arises from the uncertainty of when a given amount of money is not able to be given back on a specific date. The definition given by Bessis (2002), is the one that says it is the risk of losses associated in the event of default by the borrower, or in the event of a worsening in their credit quality. Applied to the insurance industry, it is the risk of the counterparty's default, including in the event that a reinsurer is unable to meet its commitments set out in the reinsurance contract.
- Operational Risk: This is the risk of incurring losses resulting from an inadequate internal process of people or systems that affects the business of the insurance company. It is a risk that is not specific to the life insurance industry and its quantification presents major problems due to the lack of information explaining the risk factors that cause it.
- Additional Risk: This is the risk that is inherent in the policies sold by an insurance company. Examples of these risks include changes in the behaviour of natural disasters due to climate change, changes in the mortality tables of life insurance products, or changes in the behaviour of policyholders (prepayment behaviour).
- Risk for contracting life insurance: this is the risk that arises from such contracting and is associated with actions that the insurance company takes. These risks are reflected in the setting of rates for the chosen life insurance (calculation of the premium to be paid), in the process of establishing technical provisions as well as the process of managing assets and liabilities. Among these risks for signing up life insurance are ones that are detailed in the following table below, with longevity being found in the life insurance section with survivor coverage.

Table 1. Life insurance risks

Risk of Portfolio Loss	Risk of getting a higher or lower rate of reductions, prepayments or reclamations than estimated by the company.
- From reclamation	Arises from the possibility that the policyholder contests the insurance contract.
- Reduction	Arises from the non-payment of the premium.
Biometric Risk	Rises from the uncertainty about the future behaviour of the insured party mortality on the activity and results of the company.
- Mortality Risk	Higher than expected mortality.
- Longevity Risk	Survival higher than expected.
- Disability Risk	

Source: Authors.

It is necessary that efficient management of the risks facing the company is carried out, and this requires that such risks be measured through a risk measure. In order to apply a risk measure, the probability that the damage covered in the policy takes place must be able to be measured, which in this case is the probability of survival of the policyholder/insured party.

A risk measure is a function $M:X \to [0;\infty)$ which assigns a risk X a nonnegative real number $M(X)$. This number is the additional amount which incorporates risk X in order to be accepted by the insurance company (Gómez and Sarabia, 2008). In this study, the number assigned to the random variable (in life insurance it is Residual Life) which is the net premium payable by the insured party. Therefore, setting rates can be done by employing a risk measure.

There is a link between a premium calculation principle and a risk measure. The definition of both concepts is practically the same. A risk measure is a function M which assigns a risk X a nonnegative real number, $M(X)$, (as indicated above), while a premium calculation principle is a function $H(X)$ which assigns a risk X a real number, this number being the premium. It is important to note that it has always been important to obtain the corresponding premiums that adequately reflect the uncertainty inherent in the distribution of the risk X from a coherent risk measure. Premiums should be able to be calculated from a coherent risk measure later on this basis. Afterwards, any premium calculation principle is a risk measure.

2.1. Axioms of coherence for a premium calculation principle

A criteria of coherence is one that provides economically sound risk contributions. Tasche (2000) added that such coherency criteria must be compatible with the risk-adjusted assessment, thus allowing its proper management. The criteria of coherence is related to the verification of four properties, so that any premium calculation principle that satisfies these properties will be optimal in order to carry out effective risk management, since an efficient allocation of the premium to the residual life random variable will be performed (Artznery Delbaen, 1999; Landsman and Sherris, 2001).

Listed below are the selected properties, where 'a' is a constant.

- Positive Homogeneity: $M(aX)=aM(X);\ a\geq 0$.
- Invariant to displacements: $M(X+a)=M(X)+a$.
- Monotonicity: $M(X_1)\leq M(X_2)$
- Subadditivity: $M(X_1+X_2)\leq M(X_1)+M(X_2)$

It is convenient to carry out an interpretation of each of these properties for the life insurer. One possible interpretation of the homogeneity property means that if upward, downward or modifying inflationary effects are produced in the measurement unit that affect the amount of that loss, these effects are reflected in a way that is directly proportional to the premium.

With respect to the conversions invariance property, one possible interpretation means that if the risk covered by the insurance company is seen to be increased by some external factor, thus becoming a greater risk to the company, this negative effect has to move directly in an additional way to the premium charged by the company.

An interpretation for the monotony property implies that if a company supports the risk coverage which is worse than another, logically because the of the risk that is more harmful to the company, it will have to charge a higher premium to the policyholder.

Finally, regarding the subadditivity property, its possible interpretation means that in an insurance company that has a portfolio comprised of 'n' policies, the overall risk when considering all the portfolio policies must be less or equal to the risk of considering each of the policies individually, since the risks of each policy offset one another when considering the entire portfolio.

In the following table the premium calculation principles are shown employed in actuarial literature. Only two of these the net premium principle and Wang's distortion function principle as a power make up coherent risk measures. That is why they have been selected from the rest in order to carry out the pricing on an income insurance.

Table 2. Principles for calculating premiums

P. Net premium	Yes constitutes coherent risk measure
P. Expected value	Does NOT constitute coherent risk measure
P. Variance	Does NOT constitute coherent risk measure
P.Typical deviation	Does NOT constitute coherent risk measure
P. Exponential	Does NOT constitute coherent risk measure
P.Esscher Premium	Does NOT constitute coherent risk measure
P. Wang function	Yes constitutes coherent risk measure

Source: Authors.

2.2. Performance of the coherence axioms by the selected premium calculation principles

2.2.1. Net premium principle

This is a special case of the Principle of Expected Value, where the parameter $\theta = 0$.
$H(X)=(1+\theta)E[X]$. For the value of $\theta = 0$, $H(X)=E(X)$.

This principle constitutes a coherent risk measure since it satisfies the four properties considered desirable. It is therefore considered suitable for setting rates. In fact it is the principle that is used in the area of life insurance (See example in Hernández, 2013). The problem is that it does not provide an adjusted premium in any way (either express or implied) and for this reason the insurance companies work with outdated mortality tables in order to avoid unfavourable deviations from actual claims with regards to expected claims.

2.2.2. Principle of the Wang Distortion Function as a power

$$H(X)=\int_0^{\infty} g\left(S_X(x)\right)dx=\int_0^{\infty}\left(S_X(x)\right)^{1/\rho}dx\,;\ \rho\geq 1 \qquad (1)$$

The function 'g' modifies the survival function, transforming it. Therefore, the expression $g(S_X(x))$ is known as the risk-adjusted survival function. If the parameter 'ρ' takes on the value of 1, it produces the special case of the risk measure based on the previously explained net premium principle. This principle also constitutes a coherent risk measure, since it verifies the four properties discussed above (See example in Wang, 1995). What then takes place is the setting of rates for an income insurance by using this principle, used for calculating the risk premium with an implicit surcharge to date in the non-life sector. The contribution of this work is its application to the field of life insurance and for the two survival laws mentioned in point 1.

3. Application of the net premium principle and the principle of the wang distortion function for setting rates in a survival insurance

The obtaining of a single risk premium for an insurance with survival coverage, income insurance, is to be performed from the principles of premium calculation that are considered coherent risk measures (explained in point 2). In this form of insurance the insurer is committed, at the end of a deferral period agreed in the policy, to pay the insured party and while they receive a regular income. (Bowers et al., 1997). In order to be entitled to these amounts the insured party must start paying the company the amount of the premiums, whether regular or a single premium, on the date upon signing the insurance contract. The random variable is the residual life variable or time left to live from age x, T_x .

The following cases will be considered for this type of income insurance:
- The company pays the insured party 1 u.m. while the insured remains alive.
- The technical interest rate is 'i'.
- Given the continuous random variable age of death of a newborn aged 'X', with survival function $S(x)$, random variable T_x, remaining life or time left to live from age 'x', it has a distribution function called $G_X(t)$ and a survival function $S_X(t)$.

3.1. Application of the net premium principle for calculating the single risk premium

The general expression of the premium for this type of insurance (Bowers et al., 1997):

$$\begin{aligned}
&P=\int_0^{\infty} v^t\left(1-G_x(t)\right)dt\\
&{}_tp_x=P\left(X>x+t/X>x\right)=1-G_x(t)=S_x(t) \qquad (2)\\
&P=\int_0^{\infty} v^t(1-G_x(t))dt=\int_0^{\infty} v^t\,{}_tp_x\,dt=\int_0^{\infty} v^t S_x(t)dt
\end{aligned}$$

The following change of variables is done in order to express this risk single premium in terms of the Remaining Life random variable.

If v^t=z; then tLnv=Lnz. Solving for t becomes: t=Lnz/Lnv

If $t=0$, then $v^0=1$. The variable 'z' takes on the value 1.

If $\lim_{t\to\infty} v^t=0$, the variable 'z' takes on the value 0 since the factor 'v' is less than the unit value.

Therefore we have:

$$P=\int_0^{\infty} v^t S_x(t)dt=\int_1^0 z\frac{S\left(x+\frac{Lnz}{Lnv}\right)}{S(x)}\frac{1}{zLnv}dz=-\int_0^1 zS_x\left(\frac{Lnz}{Lnv}\right)\frac{1}{zLnv}dz \qquad (3)$$

Combining the parts of the above expression (Hernández, 2013) leads to the desired expression:

$$P=-\frac{1}{Lnv}\int_0^1 S_x\left(\frac{Lnz}{Lnv}\right)dz \qquad (4)$$

Mathematical expressions of the single risk premiums for the Gompertz & Makeham survival laws are reflected in the following table. (See the mathematical developments for them in Hernández, 2013).

Table 3. Risk single premiums through the Net Premium Principle

Reset risk single premium	Gompertz law application	Makeham law application
$P=-\frac{1}{Lnv}\int_0^1 S_x\left(\frac{Lnz}{Lnv}\right)dz$	$P=\frac{-1}{g^{C^x}\left(C^{x+1}Lng+Lnv\right)}$	$P=\frac{-1}{g^{C^x}\left(LnS+C^{x+1}Lng+Lnv\right)}$

Source: Authors.

3.2. Application of the Wang distortion function principle as a power in order to implicitly calculate the recharged risk single premium

The general expression that this recharged premium has, transforming the survival function according to the expression of the equation (1) is:

$$P_{rec}=-\frac{1}{Lnv}\int_0^1\left(S_x\left(\frac{Lnz}{Lnv}\right)\right)^{1/\rho}dz,\quad \rho\geq 1 \qquad (5)$$

The mathematical expressions of the recharged risk single premiums for the Gompertz & Makeham survival laws are reflected in the following table. (See their mathematical developments in Hernández, 2013).

As can be seen, only the value of the parameters is modified when applying this premium calculation principle. But the survival law remains invariant by the fact of transforming the survival function as a power, raising it to an exponent where the considered parameter value must be greater than, or equal to, the unit in order to be considered a coherent risk measure (Wang, 1995).

Table 4. Implicitly recharged risk single premiums through the Principle of the Wang distortion function as a power

Reset risk single premium	Gompertz law application	Makeham law application
$P_{rec} = -\frac{1}{Lnv}\int_0^1 \left(S_x\left(\frac{Lnz}{Lnv}\right)\right)^{1/\rho} dz$	$P_{rec} = \frac{-1}{g^{C^x\, 1/\rho}\left(C^{x+1}\frac{1}{\rho}Lng + Lnv\right)}$	$P_{rec} = \frac{-1}{g^{C^x\, 1/\rho}\left(\frac{1}{\rho}LnS + C^{x+1}\frac{1}{\rho}Lng + Lnv\right)}$

Source: Authors.

The value of 'ρ' being greater than the unit, the instantaneous amount decreases, i.e., the policyholders live longer, causing a negative economic impact on the company in the form of survival insurance coverage, which results in the adjusted premium having to be higher, as there is more risk to the insurer.

Due to the distortion function, the probability of survival is greater (since 'S' is a number less than the unit value, raised to a number $\frac{1}{\rho}$ that is also smaller than the unit value). And considering an income insurance this is negative for the company, so that it will charge a higher premium, as if it were adjusting the survival probability.

The obtained adjusted premium corresponds to the pure premium of a new random variable 'Y' lasting until death, with an instantaneous amount for mortality proportional to the initial variable 'X' (which corresponds to the net premium principle).

$$
\begin{aligned}
\mu_X &= -LnS \\
\mu_Y &= -LnS^{\frac{1}{\rho}} = \frac{1}{\rho}(-LnS) = \frac{1}{\rho}\mu_X
\end{aligned}
\qquad (6)
$$

Analyzing the Gompertz law, the use of the Wang distortion function affects the value of the parameter 'g', which is less than the unit value, producing the same general effect, i.e., an increase in of the probability of survival, therefore resulting in a higher adjusted premium than the non-adjusted premium. But the law remains invariant, modifying only the value of the parameter 'g'. The proportionality of the amounts continues to be verified with this law.

$$\mu_x = -\mathrm{Ln}\, g\, \mathrm{Ln}\, C\, C^x$$
$$\mu_y = -\mathrm{Ln}\, g^{\frac{1}{\rho}}\, \mathrm{Ln}\, C\, C^x = \frac{1}{\rho}\left(-\mathrm{Ln}\, g\, \mathrm{Ln}\, C\, C^x\right) = \frac{1}{\rho}\mu_x \qquad (7)$$

The obtained adjusted premium coincides with the pure premium of another life expectancy variable which follows a Gompertz law of parameters C and $g^{\frac{1}{\rho}}$, being the instantaneous amount proportional to the corresponding one of the original variable 'X'.

And with regards to the Makeham law, which deals with two parameters, 'g' and 'S', both less than the unit, the effect that it has introducing an implicit adjustment , because of the use of the distortion function as a power, is exactly the same as under the previous law. The law remains invariant considering the use of the Wang distortion function. The survival law does not change, only the parameter values change. The model therefore does not change, it remains a Gompertz model or a Makeham model.

$$\mu_x = -\mathrm{Ln}S - \mathrm{Ln}g\ \mathrm{Ln}C\,C^x$$
$$\mu_y = -\mathrm{Ln}S^{\frac{1}{\rho}} - \mathrm{Ln}g^{\frac{1}{\rho}}\mathrm{Ln}C\,C^x = \frac{1}{\rho}\left(-\mathrm{Ln}S - \mathrm{Ln}g\ \mathrm{Ln}C\ C^x\right) = \frac{1}{\rho}\mu_x \qquad (8)$$

The reset premium coincides with the pure premium of another life expectancy variable that follows a Makeham law with parameters $S^{\frac{1}{\rho}}$, $g^{\frac{1}{\rho}}$ and C. In this way the instantaneous amount of mortality is proportional to the corresponding one of the original variable X, the proportionality factor being $\frac{1}{\rho}$.

The following two graphs reflect the upward trend that has the adjusted risk premium set against increases in the parameter value. In order to assign numerical values of the parameter, the instructions that SOLVENCIA II make in their QIS5 technical report to insurance companies have been taken into consideration, which indicates that the value for the survival insurance parameter will be $\rho=1.20>1$. In this article we have extended the numerical variation scope of this parameter in order to carry out a comparison, both numerically and graphically, between the adjusted risk premium and the net risk premium or not adjusted.

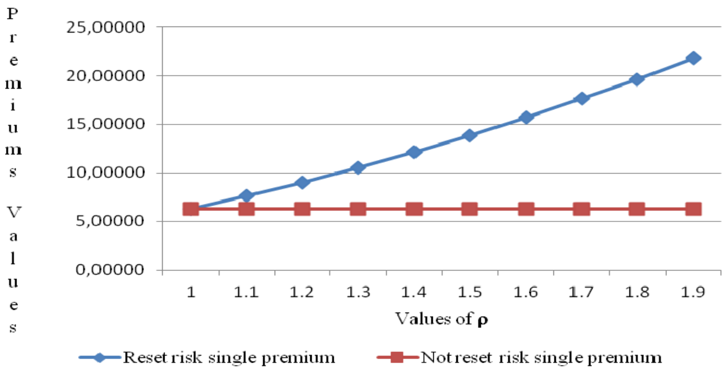
Variations of 0.1 to 0.1 have been considered until arriving at the value of 1.9. The numerical values of both premiums have been derived from the values assigned to the parameters: Actuarial age: 40 years, technical interest rate of 1%, $S=0.999$, $g=0.9969$ y $C=1.1034$. The data has been taken from the mortality tables produced by Prieto & Fernández: Table projected from the year 2000 of Spanish mortality from 1950 to 1990.

Table 5. Comparison of the reset risk and net premium for the Gompertz Law

ρ	Reset risk single premium	Not reset risk single premium
1	6.3241	6.3241
1.1	7.6113	6.3241
1.2	9.0100	6.3241
1.3	10.5183	6.3241
1.4	12.1347	6.3241
1.5	13.8573	6.3241
1.6	15.6845	6.3241
1.7	17.6147	6.3241
1.8	19.6464	6.3241
1.9	21.7779	6.3241

Source: Authors.

Graphic 1. Comparison of both premiums based on the Gompertz law



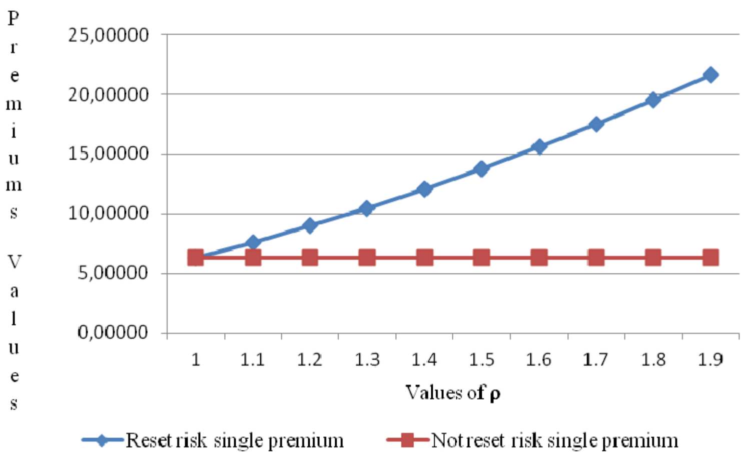
Source: Drawn by authors from Table 5. The graph shows the impact of the implicit adjustment on the pure premium according to the increase of parameter ρ , based on the Gompertz law.

Table 6. Comparison of the reset risk and net premium for the Makeham law

ρ	Reset risk single premium	Not reset risk single premium
1	6.2901	6.2901
1.1	7.5706	6.2901
1.2	8.9621	6.2901
1.3	10.4627	6.2901
1.4	12.0709	6.2901
1.5	13.7848	6.2901
1.6	15.6029	6.2901
1.7	17.5236	6.2901
1.8	19.5452	6.2901
1.9	21.6663	6.2901

Source: Authors.

Graphic 2. Comparison of both premiums based on the Makeham law



Source: Produced by authors from table 6. The graph shows the impact of the implicit adjustment on the pure premium according to the increase of the parameter ρ , based on the Makeham law.

4. Conclusions

In this article we have made a study of the calculation principles for existing premiums in the actuarial sector which constitute a coherent risk measure for verifying coherency axioms. The net premium principle is a special case of the expected value. The disadvantage is that it provides a premium without any type of adjustment and this is why insurance companies have to work with outdated mortality tables in order that the mortality they work with is less than the human group considered in the tables. On the other hand, the distortion function principle has been applied to date in the field of general insurance and this is the main contribution of the research work. In this study it is applied for the first time for the calculation of a risk single premium in an income insurance for life insurance (Hernandez, 2013). This is a principle which constitutes a coherent risk measure for the values of the parameter $\rho \geq 1$, which are the values that have to be verified in the type of insurance selected so that the reset risk premium is greater than the net premium. The contribution shown in this article is its application to the field of life insurance for an income insurance, and taking two survival laws as a calculation example. In this manner a theoretical justification has been achieved for the insurance companies' common practice of modifying the instantaneous amount of mortality.

Through the employment of the distortion function as a power, it is shown that the survival law model that is used does not change. This means that the two laws used are invariant given the use of this function, only the value of the parameters will change, as can be seen in Tables 2 & 3. Only the value of the instantaneous amount will be modified, the latter being proportional to the instantaneous amount from the survival law without distorting, and the proportionality factor being precisely the distortion function exponent, which is the ratio $^{1}\!/_{\rho}$. The usual practice in the field of life insurance of adjusting the instantaneous amount, through a coherent risk measure, is justified with this parameter ρ, and thus unfavourable deviations in claims may be dealt with.

This premium calculation principle is first applied at a general level, in order to give examples for the Gompertz & Makeham survival laws afterwards.

In this type of insurance, as the ratio $^{1}\!/_{\rho}$ is proportional to the instantaneous mortality amount, as ρ increases then the probability of death decreases, this later implies a greater risk for the insurance company. Therefore, the company will charge ever-increasing implicitly reset premiums in view of increments in the value of that parameter.

It is shown that the adjusted premium obtained for the two survival laws used in the income insurance is the same as the one derived from another random variable, which follows the same survival law, only modifying the parameters of the laws and leaving the proportional instantaneous amounts of mortality, the distortion function exponent being the proportionality factor. When working with survival coverage insurance, the distortion function exponent which is already defined as the proportionality factor, has to be less than one, thus causing the longevity risk to be greater through the use of this distortion function. For the values that take on the parameter $\rho \geq 1$, it was already shown (Wang, 1995) that this risk measure is coherent, by verifying the property of subadditivity, since the rest of the properties only require that the parameter value be $\rho > 0$. This way, one manages to get an implicitly reset risk premium higher than the net premium, having the risk single premium adjusted and the proportionality factor being a decreasing ratio.

References

Artzner, P. and Delbaen, F. (1999). Application of coherent risk measures to capital requirements in insurance. *North American Actuarial Journal*, *3*(2), 11-15.

Bessis, J. (2002). *Risk management in banking* (2nd ed.). Chichester: John Wiley and Sons.

Bowers, JR., Newton, L., Gerber, H. and Jones, D. (1997). *Actuarial mathematics*. Illinois: The Society of Actuaries.

European Commission. Internal Market and Services DG. Insurance and pensions. Brussels., (2010), QIS5 Technical Specifications (Working Document of the Commission services). https://www.ceiops.eu.

Gómez Deniz, E. and Sarabia, JM. (2008). *Credibility theory. Development and applications in insurance premiums and operational risks.* Madrid: MAPFRE Foundation.

Hernández Solís, M. (2013). *Life insurance pricing with distorted life expectancy risk measurement*. Unpublished doctoral dissertaion. Complutense University, Faculty of Economics and Business, Madrid.

Hernández, M., Lozano, C. and Vilar, J.L. (submitted). A note on life insurance ratemaking with proportional hazard transform. *Annals if the Institute of Spanish Actuaries*.

Landsman, Z. and Sherris, M. (2001). Risk measures and insurance premium principles. *Insurance: Mathematics & Economics*, *29*, 103-115.

Modigliani, M. and Miller, M. (1958). The cost of capital, corporate finance and the theory of investment. *The American Economic Review*, *48*(3), 261-297.

Prieto Pérez, E. and Fernández Plasencia, J. (2000). Mortality tables for the population of spain from 1950 to 1990. Table projected to 2000.

Tasche, D. (2000). Risk contributions and performance measurement. [Online]. Technical University of Munich.

Tse, Y.K. (2009). *Nonlife actuarial models. Theory, methods and evaluation*. Cambridge (UK): Cambridge University Press.

Wang, S. (1995). Insurance pricing and increased limits ratemaking by proportional hazards transforms. *Insurance, Mathematics & Economics*, *17*, 43-54.