Genetic Algorithm Optimization of Lift Distribution in Subsonic Low-Range Designs

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Abstract. The optimization of the lift distribution is an essential analysis in the wing design segment of every aircraft project. Although it has been demonstrated that the optimal solution follows an elliptic distribution, there is no known relation between the parameters that define this distribution and its similarity to the elliptical one. Therefore, there is no direct approach for obtaining an exact solution, existing methodologies such as CFD simulations which require of a considerable amount of time and resources to offer accurate results. The methodology followed throughout this paper involves the application of metaheuristic techniques, such as genetic algorithms, in order to optimize the lift distribution obtained through the Prandtl lifting-line theory. Results show that the genetic algorithm proposed is able to obtain a satisfactory solution within a reasonable time.

Keywords: Genetic algorithm · Lift distribution · Wing design · Elliptical lift distribution

1 Introduction

Wing design stands as one of the most crucial analysis in every aircraft project, being the main contributor to the force that lifts the aircraft as well as playing a decisive role in the efficiency of the plane. Hence, it is critical that the wings provide the amount of lift required without deriving in other negative effects such as aerodynamic resistance, stall inception and lesser fuel capacity among others.

Therefore, companies undergoing the development of a new aircraft invest a substantial amount of resources for the R+D+i of the wing design especially the long-range models. Besides, due to the concurrent engineering fundamentals [1],

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the delay of a specific section of a project, such as wing design, may cause major consequences in other departments to the point of a complete setback of the project.

Moreover, the research and development of a specific airfoil is a rather demanding project, requiring severe research in both CFD (Computer Fluid Dynamics) simulations [2] and empirical experiments like wind tunnel testing [3]. Requiring these simulations of an extensive amount of time and resources to execute.

One of the most decisive analysis of the wing design is the optimization of the lift distribution. In an ordinary wing, the lift output usually does not remain constant and it varies from the distance from the root of the wing, due to the existence of variables such as the taper ratio λ, torsion angle α_t and the wing incidence α_{set} [4]. Hence, the lift output of every section of the wing varies, creating a lift distribution. It is concluded from multiple investigations that the optimal lift distribution is the elliptic one [5,6], and every deviation from this distribution result in negative consequences such as an increase in fuel consumption, or even develop the stall phenomenon [7] and its undesired consequences.

However, the optimization of this desired result is not easily achieved, being no known relation that could be drawn between the parameters that define the lift distribution of a wing and its similarity to an elliptical distribution. As a consequence, there is no direct approach available that could be used for obtaining an exact solution for this problem.

Nonetheless, the aeronautic industry have developed a series of methodologies [8,9] that could potentially obtain an exact solution. However, these techniques rely heavily on CFD simulations, which require of a considerable amount of resources when searching for a precise solution.

On the other hand, there are other techniques which do not require of CFD simulations and offer an approximated result [10], implementing numerical methods. However, the results of these methodologies may vary depending on the initial conditions of the problem.

In the endeavor to pursue a finer solution, we propose the application of metaheuristic techniques, such as genetic algorithms, as for finding a solution of this problem that does not rely on expensive simulations.

In the previous years, we have observed the rise of these methodologies over various disciplines, from economics and decision making [11] to driving optimization [12], positioning systems [13,14] and even aerodynamics in other aspects of wing design [15]. Hence, we propose the application of this algorithm in this particular problem with the intent of obtaining the combinations of parameters that optimizes the lift distribution of our wing in a reasonable time.

2 Description of the Problem

The wings are the main source of lift in an aircraft, this force is generated from the pressure difference from the static pressure in between the upper and lower surfaces of the airfoil as air flows through it, thus generating a force that pushes the wing upwards. The amount of force generated is heavily dependent on the geometry of the airfoil and does not remain constant along the chord or length of the airfoil.

When analysing the performance of an airfoil, it is preferred the term of lift coefficient of the airfoil C_l over its force of lift, which allow us to exclude all the environmental parameters out of the equation and adimensionalizate it by the airfoil's chord. This lift coefficient can be calculated in empirical test such as wind tunnels.

$$C_l = \frac{l}{qc} \tag{1}$$

where l is the lift force, q is the dynamic pressure and c is the chord of the airfoil.

The Eq. (1) provide the lift coefficient of an airfoil, a section of the wing, so in order to obtain the total lift coefficient of the wing C_L, more additional parameters are required as rarely the airfoil of a wing remains constant.

Therefore, given the airfoil in the root of the wing, in this case the NACA 23024, it is possible to define the shape of our wing as a function of a series of parameters, such as the wing surface S, the aspect ratio AR, the taper ratio λ, the twist angle α_t and the wing incidence α_{set}.

The aspect ratio, along the wing surface, provides the scope of the wing, and it is defined as the wingspan of the wing squared divided by the wing surface.

The taper ratio indicates the narrowing of the wing from root to tip. This narrowing serves multiple motives but mainly structural ones. Although its value depends on the project's specifications, we can obtain its value by dividing the chord's length at the tip by the chord's length at the root.

As for the twist angle, this parameter indicates the deviation of the angle of attack along the wingspan. The angle of attack of a wing is the angle formed between the mean aerodynamic chord of an airfoil and the incident flow. There is a direct relation between the angle of attack and the lift generated, however, over a certain value which depends on the airfoil, the airfoil no longer generates lift, knowing this phenomenon as stall [16]. The twist angle serves as a way to prevent this event from happening as well as adjusting the lift distribution to obtain its optimized value.

Finally, the wing incidence is the angle formed between the fuselage center line and the main aerodynamic chord. This parameter allows the wing to have a higher angle of attack above all, increasing the lift budget but compromising the stall of the wing.

All these parameters are the responsible for causing an irregular lift distribution along the wingspan, which usually tends to decrease from the distance from the root, mainly for structural purposes. Although there are multiple methodologies for obtaining this lift distribution, one of the most expanded and well rounded techniques is the Prandtl Lifting-Line Theory [17] from which we can obtain the value of the wing distribution. Despite being a traditional theory, it is still being used and codified in CFD simulations [18].

In conclusion, thanks to Prandtl's theory, it is possible to obtain the lift distribution of a wing as a function of the wing surface S, the aspect ratio AR, the taper ratio λ, the twist angle α_t and the wing incidence α_{set} as well as other aerodynamic parameters linked to the airfoil selected.

$$C_{L_\alpha} = \frac{4b\,\mu}{\bar{c}} \tag{2}$$

$$\mu = \frac{1}{\alpha_0 - \alpha} \cdot \sum_{n=1}^{N} A_n \sin(n\theta) \left(1 + \frac{\mu\, n}{\sin(\theta)}\right) \tag{3}$$

where b is the wingspan, $\bar{c}$ is the main aerodynamic chord, θ the polar coordinates, n the discretization, α the segment's angle of attack, α_0 the zero-lift angle of attack and A_n the coefficients of each point.

Following the Eqs. (2, 3) obtained from Prandtl's theory, it is possible to plot the lift distribution of a certain wing. As multiple studies have proved before [19], the optimal lift distribution of any sub-sonic wing design is always the elliptic distribution. Any deviation from this optimal distribution shall derive in undesired consequences such as an increase in the aerodynamic resistance, thus an increase in fuel consumption [20].

Nonetheless, there is no direct relation which could be drawn between these aerodynamic parameters and the likeness of the lift coefficient function to the ellipse distribution. Likewise, the most expanded methodology [8,9] to confront this problem relies on assumptions such as incompressible flow which is only valid on considerable low speed scenario. Besides, these approximations usually require a great deal of simulations in CFD software and real life experiments such as wind tunnels, increasing the global cost of the project.

Hence, we propose a different approach, relying on the application of heuristic algorithms such as genetic algorithms, as a way to achieve a more adequate solution than traditional methods.

3 Genetic Algorithm

Therefore, as a consequence of the lack of a viable exact solution that does not require the assumption of unfeasible conditions or the execution of laborious CFD simulations, we propose to approach this problem with metaheuristic methodologies. Although there are multiple algorithms that could prove suitable for this problematic situation, we propose the application of genetic algorithms as a result of their exploration and solution intensifying capabilities.

We have also observed the rise of genetic algorithms optimizations over the last years in a variety of disciplines, from economics and decision making [11], to optimizing driving routes [12], positioning [14] and even aerodynamic designs [15]. Therefore, their application to this problems seems feasible.

The genetic algorithm we propose will carry the parameters that defines the lift distribution, being these the aspect ratio, the taper ratio, the twist angle and the wing incidence. However, in this paper we are studying the lift distribution of a low range subsonic aircraft [21], hence not every value of these parameters can be considered acceptable. We can determinate from the design specifications as well as other similar projects that the parameters must be within a certain region, showed in Table 1.

Table 1. Parameters from the wing design

GA wing parameters			Aerodynamic constants	
Parameter	Max value	Min value	Parameter	Value
AR	13	11	S	$6.22\ \mathrm{m}^2$
λ	0.7	0.3	α_0*	$-1.25\,\mathrm{rad}$
α_t	-3°	-1°	$\alpha_{2\pi}$*	$2\pi\,\mathrm{rad}$
α_{set}	3°	0°		
**Values obtained from airfoil NACA 23024*				

Furthermore, the proposed algorithm would carry all these variables in each and every individual of the population, coded in binary. From the difference in the range of these parameters we have created different length arrays for each variable, with a criteria for separating the digits from the whole number to the decimal part, as well as if it has a negative or positive value.

$$\alpha_{\mathrm{set}} = \underbrace{1}_{\text{sign}} \quad \underbrace{010}_{\text{whole number}} \quad \underbrace{0110101101}_{\text{decimal number}} = 2.419^\circ$$

These parameters define the lift distribution, hence, in order to optimize this distribution we must search the combination of parameters that generates the most likeness to the elliptical one. As a result, we can build a fitness function based on the difference of the lift distribution generated from these parameters and the optimal ellipse. It is possible to compute this difference with the MAE (Mean Absolute Error) or the RMSE (Root Mean Square Error).

The MAE is considered among some authors as generally the best method for evaluating a model performance [22,23], being the preferred methodology for evaluating uniform error distributions, nonetheless is a well rounded valid method.

On the other hand, the RMSE proves a better performance in normal error distributions, however, the bigger difference from the MAE is that the RMSE penalizes heavily large errors that deviate from the standard value [24].

Although both methodologies would prove suitable for this problem, the best approach is the RMSE, for a large singular error deviation may be less desirable than a low uniformed error distribution.

However, certain parameters such as the aspect ratio AR or surface of the wing S will define the dimensions of the wing, thus the scope of the lift distribution. Hence, the scope of the ellipse used to measure the elliptical likeness of the current lift distribution shall display similar dimensions with it. As a consequence, a new ellipse will be generated with each individual of the genetic algorithm.

Thence, it is possible to obtain the coordinates of the ellipse desired by adapting the ellipse equation so that it contains the lift coefficients at the root and the wingspan of the wing as they represent the intersection of the ellipse with the 2-D axis.

$$y_{\text{Ellipse}} = \sqrt{\left(1 - \frac{x^2}{C_{L_{\text{root}}}^2}\right)\frac{b^2}{2}} \tag{4}$$

where x is the discretization of the wing, b is the wingspan and $C_{L_{\text{root}}}$ the value of the lift coefficient at the root of the wing.

Nonetheless, following this approach, a more sizeable lift distribution might present a bigger RMSE than a smaller one due to its actual dimensions, even if it presents a much more suited likeness to the proposed ellipse. Still, this impediment could be easily arranged by adimensionalizating the RMSE, dividing it by the maximum value of the ellipse.

Furthermore, it is important to clarify that not every combination of these aerodynamic parameters is acceptable. Depending on the specifications of the aircraft project, these parameters should stay within certain limits. As a solution for this issue, we have created a correction factor κ which is a function of all these parameters, being its value bigger the farthest a variable stray from its expected value and null when it stays within the range specified in Table 1. Hence, the final value of κ would be added to the RMSE of the likeness of the lift distribution in order to penalize extreme and unfeasible combinations.

For the calculation of κ, we propose the following equations:

$$\kappa_{AR} = \max\left(1, \frac{|AR - AR_{\max}|}{|AR_{\max} - AR_{\min}|}, \frac{|AR - AR_{\min}|}{|AR_{\max} - AR_{\min}|}\right) \tag{5}$$

$$\vdots$$

$$\kappa = (4 - \kappa_{AR} - \kappa_{\lambda} - \kappa_{\alpha_t} - \kappa_{\alpha_{\text{set}}}) \cdot \varepsilon \tag{6}$$

where $AR_{\max}$ and $AR_{\min}$ are the maximum and minimum values of the interval AR specified in Table 1, and ε is the coefficient whose purpose is to determine the intensity of the κ penalization

Therefore, we can propose the following fitness functions, with MAE and RMSE error evaluation.

$$\text{ff}_{\text{RMSE}} = \frac{1}{C_{L_{\text{root}}}}\sqrt{\frac{\sum_{i=1}^{n}(y_{C_{L\alpha}} - y_{\text{Ellipse}})^2}{n}} + \kappa \tag{7}$$

$$\text{ff}_{\text{MAE}} = \frac{1}{C_{L_{\text{root}}}}\frac{\sum_{i=1}^{n}|y_{C_{L\alpha}} - y_{\text{Ellipse}}|}{n} + \kappa \tag{8}$$

Lastly, the algorithm shall rely on tournament based selection methodology [25] with 3 competing individuals. On the other hand, for the crossover methodology, we have implemented a multipoint based crossover. Likewise, the algorithms shall operate with small percentages of elitism and mutation, deduced in base of the results of previous simulations (Table 2).

Table 2. Genetic algorithm configuration

GA	Selection
Population size	60
Selection technique	Tournament 3
Elitism	5%
Crossover	Multi-point
Mutation	3%
Convergence criteria	50 generations or 80% individual equals
ε data validation	$5 \cdot 10^{-3}$

4 Results

Once set up and executed in the Python programming language, the algorithm showed a rapid convergence to an acceptable solution in a short interval of time. Due to the circumstances of this problem, a limited population had sufficed to reach the desired solution in an adequate number of generations, proving that this method could be considered as a viable alternative over long and resource-heavy CFD simulations. Therefore, the genetic algorithm proposed have obtained the following solution:

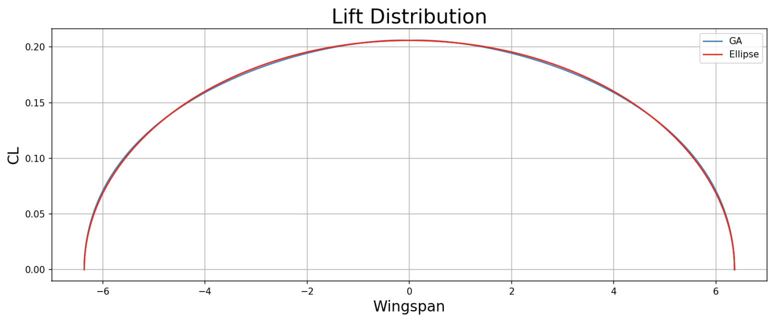

Fig. 1. Lift Distribution provided by GA. The blue curve represents the lift distribution through the wingspan (meters), provided by the RMSE variation of the genetic algorithm

As shown in Fig. 1, the lift distribution provided by the genetic algorithm proves a convenient likeness to the elliptical distribution desired, proving the suitability of this methodology.

Figure 2 shows the evolution of the RMSE along generations, thus we can appreciate the accelerated convergence to the final solution within a couple generations (Table 3).

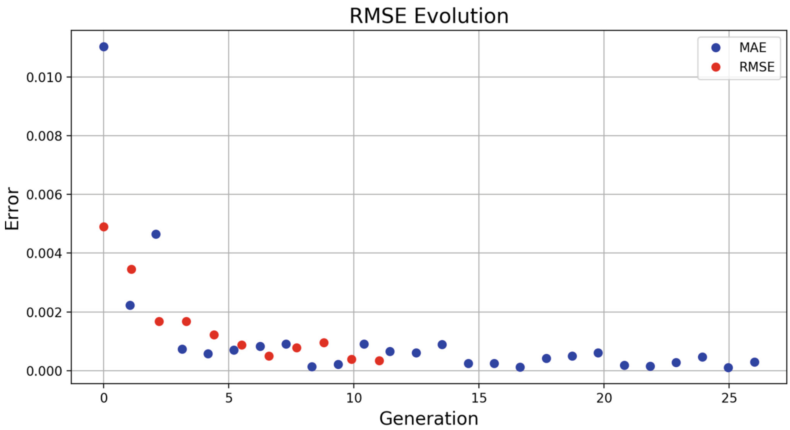

Fig. 2. Genetic Algorithm's lowest error for every generation with RMSE and MAE adaptations. The RMSE variation converged in generation 11 unlike the MAE where the convergence criteria was fulfilled in generation 26

Table 3. Results of the Genetic Algorithm

	RMSE variation	MAE variation
AR	11.5	10.8
α_t	$-1.738°$	$-1.684°$
α_{set}	$0.403°$	$0.234°$
λ	0.817	0.832
Lowest error	$4.949 \cdot 10^{-4}$	$\mathbf{3.88 \cdot 10^{-4}}$

Both variations of the genetic algorithm have proven to be satisfactory. The MAE variation showed a lower error in the best individual but the RMSE was rather stable and had a faster convergence.

5 Conclusion

Wing design represents a substantial analysis in every aircraft project, being one of the fields with the largest amount of resources invested in. One of the most important steps of the wing design is the optimization of the lift distribution, as the airfoil of the wing usually suffer a deviation from its original form in the root. It is concluded that the optimized lift distribution is the elliptical one, thus every deviation from this ideal distribution will result in undesired consequences such as an increase in fuel consumption.

However, there is no know relation between the aerodynamic parameters that define the wing and the likeness of the lift distribution to an ellipse. This problem has been confronted by numerous methodologies, from CFD computer simulations that could provide an exact solution, thought requiring of a considerable amount of time and resources to execute, to numerical methods that offer a close approximation.

In this paper we have proposed the application of metaheuristic techniques such as genetic algorithms to confront this problem in the pursue of an acceptable solution that does not require of any laborious simulations. We have discussed the different approaches for constructing the genetic algorithms with multiple fitness functions and we have made the adjustments required.

Results show that the genetic algorithm proposed is able to reach a robust solution in a reasonable time with both fitness functions designed, being thus fulfilled the main objective of this paper.

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