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Rescaling and Transforming Continuous Non-Normal Data in Covariance Structure Analysis: An Essay of Taxonomy

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Re-escalamiento y transformación de datos continuos no normales en análisis de estructuras de covarianzas: un intento de taxonomía

Resumen. A partir de datos recopilados de una investigación previa con modelos de Análisis de Estructuras de Covarianzas, este artículo pretende clasificar los distintos tratamientos y transformaciones necesarios para variables observables con datos continuos y ordinales considerados como no-normales.

Palabras clave: datos normales, modelización con estructuras de covarianzas, métodos ML, GLS, AGLS, robusto, matriz de correlaciones de Pearson, transformaciones poliseriales y policóricas.

Abstract. Based on data compiled from previous research with Covariance-based Structural Equation Models, this paper seeks to introduce a classification of different treatments as well as necessary transformations for observable variables with continuous and ordinal data considered as non-normal.

Key words: normal data. Covariance Structural Modeling. ML, GLS, AGLS, robust optimization methods. Pearson correlation matrix, polyserial and polychoric transformations.

Introduction

The main objective of this paper is to present various ways existing of dealing with non-normal data presented on a continuous scale, as well as their transformation to adjust models (Saris, 1982, Bentler and Yuan, 1998).

The starting point of this article is the discussion of actual methodology used in many papers when data are not considered as normal. The unique method offered to researchers is that of transforming the Pearson correlation matrix in another matrix with polyserial and/or polychoric partition (Joreskog, 1986). In a second stage it is suggested to submit the new matrix to the Asymptotic General Least

Squares (AGLS) (Bentler and Wu, 2002) or Asymptotically Distribution Free procedure (Steiger, 1990; and Steiger and Hakstian, 1982) depending on what program is considered (Sharma *et al.*, 1989).

Data issued from questionnaires are traditionally ordinal. When they are not normally distributed it is suggested (or more exactly the Pearson correlation matrix) that they be submitted to some programs like Prelis (Pre-processor to Lisrel) (Joreskog 1990) or EQS (Bentler, 1993), which transform the correlation matrix into a new polyserial and polychoric correlation matrix. The polychoric partition computes new thresholds and recenters data, making (normality adjustments) them more adjustable to traditional covariance structural programs.

Traditionally data are presented on an ordinal scale, which facilitates the task. In situations where data are presented on a continuous scale, the consequence is an extended scale and an increase possibility that the data be non-normal (Bagozzi, 1982; Bentler and Weeks, 1980, Lee *et al.*, 1995).

In this paper we will submit data that are highly non-normal but nevertheless adjusted to more traditional methods

(McDonald, 1984) and fitting well. We will proceed transforming data, scales and matrices for a better fit (Mardia, 1970).

Data are issued from a large questionnaire (Lévy Mangin 1997, 1999a) and presented in a continuous scale ranging from 1, which represents the lowest score to 100, the maximum score.

Initially, the data will be analyzed as continuous, and fit indices will be computed according to Maximum Likelihood (ML), General Least Squares (GLS) and Asymptotic Distribution Free or Asymptotic General Least Squares (AGLS) procedures (Browne, 1984; Arbuckle, 1995; Bentler and Yuan, 1998 and 1999).

In a second stage data will be partly rescaled whereby some will be kept as continuous while the rest is rescaled within a range of 1 to 5 instead 1 to 100. The new data will then be submitted to the procedures defined in stage 1.

In a third stage, the partly rescaled data will be transformed so that the Pearson correlation matrix will be converted into a polyserial-polychoric matrix and submitted to four classical procedures ML, GLS, AGLS and the Satorra-Bentler Robust procedure.

In a fourth stage all data be completely rescaled, fitted and analyzed according to the four traditional procedures of Structural Equation Modeling.

The last stage will consist of transforming the complete rescaled data into a polychoric correlation matrix, which will be submitted to the four classical optimization procedures.

1. Implementation

To implement a taxonomy, an example of non-normal data has been selected with 99 data and eight variables (b2 to f3) (Lévy Mangin, 1997, 1999a y 1999b) (see table 1). In the next table we can observe that variables f3, f1, d3, b4 and b2 have important Skewness coefficients with critical ratios higher to -1.96. The Kurtosis critical ratios appear to be within two boundaries -1.96 to 1.96, excluding f1.

Assessment of normality						
	min	max	skew	c.r.	kurtosis	c.r1
f3	20,000	99,000	-0.737	-2.994	0.429	0.871
f1	10,000	100,000	-0.782	-3.177	1.286	2.612
d6	10,000	100,000	-0.251	-1.019	-0.295	-0.600
d3	0,000	100,000	-0.522	-2.121	0.841	1.709
e 1	10,000	100,000	-0.364	-1.480	-0.536	-1.090
c1	0,000	100,000	-0.483	-1.961	-0.125	-0.253
b4	0,000	100,000	-0.593	-2.410	0.691	1.403
b2	30,000	100,000	-0.824	-3.348	0.691	1.403
fultivariate					52,114	20.497

Table 2. Fit for continuous data with ML, GLS and AGLS procedures (Chi-					
square Probability).					
Procedure	ML	GLS	AGLS		
Probability	0.256	0.357	0.299		
All these three procedures are very significant for 1- alpha = 0.05.					

Data are issued from a questionnaire, which was administered in a prior research. The model used could be a factor analysis with three latent variables: the Product Satisfaction (PS) with four indicators b2, b4, c1 and e1; Rebates (R) with two indicators d3 and d6; Satisfaction of Performance (SP) with two indicators f1 and f3.

1.1. Adjusting continuous data

As previously explained, the variables data will be presented in a continuous scale ranging from 1, the lowest score, to 100, the maximum score. All variables will be ranged on the same scale.

The first adjustments have been made according to the three commonly used procedures, ML (Maximum Likelihood), GLS (General Least Squares) and AGLS (Asymptotic GLS) used for non-normal data.

The sample size is 99 and the chi-square fit measure will be that we will analyse. The probability will be analyzed on a scale ranging from zero to 1, with zero representing a bad fit and 1 representing a perfect fit (a Chi-square probability > 0.05 means a good fit). See table 2.

1.1.1. Partly rescaled data

In stage 2 of the procedure, the data have been partly rescaled: variables b2 to d6 are still continuous on a scale of 1 to 100 while f1 and f3 become ordinal along the new scale of 1 (the lowest score) to 5 (the maximum score. This new scale is based on subjective criteria: scores between 1 to 20 on the old scale corresponds to 1; scores of 21 to 40 corresponds to 2; scores of 41 to 60 correspond to 3 on the new scale; scores of 61 to 80 relate to 4 on the new scale while 81 to 100 relate to the maximum score of 5 of the new scale.

Rescaled data are more homogeneous and variable variances should be narrower.

The new partly rescaled data have been fitted with the three traditional procedures, ML, GLS and AGLS (see table 3).

1.1.2. Partly rescaled data with polyserial and polychoric partition With the new partly rescaled data, we can still observe three non-normal variables, the Skewness Student's Test

Table 3. Fit for partly rescaled data with ML, GLS and AGLS procedures (Chi-square					
Probability).					
Procedure	ML	GLS	AGLS		
Probability	0.245	0.317	0.359		
All these three proces	dures are very signif	icant for 1- alpha = 0	,05.		

		Asses	ssment of norn	nality		
	Min	max	skew	c.r.	kurtosis	$c.r^1$
f3	20.000	99.000	-0.737	-2.994	0.429	0.871
f1	10.000	100.000	-0.782	-3.177	1.286	2.612
d6	1.000	5.000	0.141	0.571	-0.210	-0.427
d3	1.000	5.000	0.142	0.578	-0.346	-0.703
e 1	1.000	5.000	-0.184	-0.747	-0.796	-1.616
c1	1.000	5.000	0.014	0.058	-0.693	-1.408
b4	1.000	5.000	0.121	0.490	-0.313	-0.635
b2	1.000	5.000	-0.560	-2.277	0.264	0.536
Multivariate					37.430	14.721

Results of Polyserial Partition Using V 6 - 5 Categories					
Thresholds					
	Estimates		STD. ERR		
	-1.1663		0.1596		
	-0.3852		0.1229		
	0.4055		0.1243		
	1.6174		0.2126		
	Estimates				
Variable	Covariance	STD. ERR	Correlation	STD. ERR	
V7	0.3493	0.0901	0.3493	0.0901	
V8	0.3738	0.0877	0.3738	0.0877	

		Resu	ılts of polychor	ic partition		
			Average thres	holds		
	V 1	-1.8689	-1.1617	-0.1450	1.2206	
	V 2	-1.6166	-0.4701	0.6334	1.6549	
	V 3	-1.5421	-0.4369	0.4026	1.4724	
	V 4	-1.4455	-0.2311	0.8679	2.0923	
	V 5	-1.3138	-0.2551	0.9023	1.9190	
	V 6	-1.1671	-0.4020	0.4034	1.6342	
	Poly	choric correlati	on matrix bety	ween discrete v	ariables	
	V 1	V 2	V 3	V 4	V 5	V6
V 1	1.000					
V 2	0.507	1.000				
V 3	0.466	0.494	1.000			
V 4	0.453	0.495	0.537	1.000		
V 5	0.413	0.433	0.481	0.664	1.000	
V 6	0.630	0.485	0.642	0.568	0.623	1.000

for f1 (-2.99), f3 (-3.17) and b2 (2.61) are non-normal. Skewness are superior to + or -1.96. Moreover, the Student's Test for f1 Kurtosis shows also a significant nonnormal treshold for this variable.

In order to reestablish equilibrium and re-centre the Pearson correlation matrix, a polyserial and a polychoric transformations of the correlation matrix has been performed (table 5).

This transformation is achieved as the program computes four objective thresholds for each variable (in this case V6 or d6) with estimates and standard errors corresponding to polyserial partition of the two continuous variables V7 and V8 (f1 and f3). Information on the polyserial correlation

> is presented first in the printout (before polychoric information) and is then followed by estimates for polyserial correlation with the categorical variable (here V6 or d6) computed for continuous variables (V7 and V8 or f1 and f3).

> A similar process is done for each categorical variable (V1 to V6) before yielding information on polychoric correlation estimates.

> In a second stage involving this part of the printout, we will find average thresholds after a polychoric partition of the Pearson correlation matrix for variables V1 to V6 or b2 to d6, and then a polychoric correlation matrix between discrete variables will follow.

> In this case the program has computed four average thresholds for variables V1 to V6 (b2 to d6) and shows the polychoric correlation matrix between ordinal variables (table 6).

> In table 7 we may observe that the significant procedures for a probability of 1 - alpha > 0.05 are the Robust, the AGLS and the GLS procedures.

1.2. Adjusting ordinal or completely rescaled data

In this section continuous data have been completely rescaled to become ordinal data. All variables have changed: b2 to d6 and f1 to f3. The new scale which is based on subjective criteria is as follows: scores of 1 to 20 from the old scale correspond to a score of 1 on the new scale,

scores of 21 to 40 correspond to a score of 2, scores of 41 to 60 correspond to a score of 3, scores of 61 to 80 correspond to a new score of 4 and scores of 81 to 100 relate to the maximum score of 5.

The new data appear in order and variables could practically be considered as normal (the Skewness of b2 is slightly non normal, -2.277).

1.2.1. Rescaled ordinal data

Table 9 will show the different fits for rescaled ordinal data with the four classical methods of estimation. All these three procedures are very significant for a probability of 1 - alpha > 0.05.

The GLS estimation procedure is the most significant procedure followed by that of the AGLS and ML, with little difference between the probabilities of the three Chi-square estimation methods. Adjustments of the com-

pletely rescaled data (ordinal) shown in table 4 for the three procedures (ML, GLS, AGLS) show that the rescaling of data has improved the model.

1.2.2. Rescaled ordinal data with polychoric partition

As a consequence of the rescaling the data to ordinal, these data have now become quite normal. In the event that this could not be achieved, it is possible in a first stage to perform a polychoric transformation of the Pearson correlation matrix and then submit it to the AGLS procedure in a second stage.

In this case, data are ordinal and a polychoric transformation of the Pearson correlation matrix could be performed, the printout will show estimates of four thresholds (average and non estimates) as well as the polychoric correlation matrix between ordinal variables.

This polychoric matrix then will be submitted to the AGLS procedure. Even if the data reveal to still be non-normal, this procedure should adjust well the model.

For small samples it is suggested that the Yuan-Bentler statistics (Bentler and Yuan, 1998 and 1999) be used.

The Robust procedure is the most significant for a probability of

1 - alpha > 0.05, followed by the AGLS pro-cedure (GLS and ML are also significant but at a lowest level). See table 11.

2. Discussion

The essay of taxonomy will attempt to classify treatments and methods and choosing among them, those, which are the best according some criteria that will be define.

The first criterion will be the best adjustment of data, however we realize that the sample size is limited and some caution should be exercised in regard to any generalization.

Table 7. Fit for partly rescaled data with polychoric and polyserial partition with ML GLS, AGLS and the Robust-Satorra Bentler procedures (Chi-square Probability).

Procedure ML GLS AGLS Robust Probability 0.0271 0.0609 0.75745 0.9999

In the table 3 we may observe that the significant procedures for a probability of

1 - alpha > 0.05 are the Robust, the AGLS and the GLS procedures

Table 8. Normality tests for completely rescaled ordinal data and Critical Ratios (Student's t-Test) Assessment of normality min kurtosis c.r. f3 5.000 1.000 -0.325 -1 320 0.203 0.412 f1 1.000 5.000 -0.080 -0.3240.302 0.613 d6 5.000 1 000 0.141 0.571 -0.210-0.427d3 1.000 5.000 0.142 0.578 -0.346 -0.703 e1 1.000 5.000 -0.184-0.747-0.796-1.616 c1 1.000 5.000 0.014 0.058 -0.693-1.408b4 1.000 5.000 0.121 0.490-0.635 -0.313h2 1.000 5.000 -0.560 -2.277 0.264 0.536 Multivariate 27.440 10.792

			Resu	lts of polycl	noric partiti	on		
				Average th	resholds			
		V 1	-1.6402	-1.0168	-0.1252	1.0668		
		V 2	-1.4172	-0.4096	0.5534	1.4477		
		V 3	-1.3512	-0.3821	0.3538	1.2874		
		V 4	-1.2667	-0.2015	0.7601	1.8275		
		V 5	-1.1523	-0.2222	0.7913	1.6737		
		V 6	-1.0205	-0.3521	0.3523	1.4311		
		V 7	-1.6365	-0.7548	0.4998	1.5413		
		V 8	-1.6272	-0.8333	0.3522	1.6444		
		Polychoric	correlation	n matrix be	tween discr	ete variable:	s	
	V 1	V 2	V 3	V 4	V 5	V 6	V 7	V 8
V 1	1.000							
V 2	0.507	1.000						
V 3	0.466	0.494	1.000					
V 4	0.453	0.495	0.537	1.000				
V 5	0.413	0.433	0.481	0.664	1.000			
V 6	0.630	0.485	0.642	0.568	0.623	1.000		
V 7	0.396	0.452	0.337	0.317	0.341	0.416	1.000	
V 8	0.350	0.371	0.426	0.337	0.261	0.414	0.812	1.000

Table 9. Fit for complete	ely rescaled ordinal data with	ML, GLS, AGLS procedures	(Chi-square Probability).		
Procedure	ML	GLS	AGLS		
Probability	0.547	0.609	0.571		
All these three procedures are very significant for 1 - alpha = 0.05.					

Table 11. Fit for completely rescaled ordinal data with polychoric partition with ML, GLS, AGLS and Robust procedures (Chi-square Probability).

Procedure	ML	GLS	AGLS	Robust
Probability	0.04964	0.10776	0.35319	1.000

The Robust procedure is the most significant for a probability of $\ 1$ -alpha >0.05followed by the AGLS procedure (GLS and ML are also significant but at a lowest

Table 12. Fit for all models with ML, GLS and AGLS procedures (Chi-square Probability). Procedure Data and treatment GLS AGLS ROBUST 0.256 0.357 0.299 1. Continuous data 2. Partly rescaled continuous data 0.245 0.317 0.359 3. Partly rescaled continuous data to ordinal with polyserial and polychoric transformation 0.027 0.060 0.757 0.9999 4. Completely rescaled continuous data to ordinal 0.547 0.609 0.571 5. Completely rescale continuous data to ordinal 0.049 0.107 0.353 1.0000 data with polychoric transformation

Table 13. Yuan Bentler Statistics tests with the AGLS Procedure (Chi-square Probability and F probability).					
Corrected Statistic for each Model	3. Partly rescaled continuous data with polyserial and polychoric transformation	5. Completely rescaled ordinal data with polychoric transformation			
Yuan-Bentler corrected c2	0.84388	0.54922			
Yuan-Bentler corrected					
F-Statistic	0.86360	0.55981			

With only 99 valid cases we think that the best-fit index would be essentially the Chi-square and its probability of occurrence.

We can observe that continuous non-normal data fit pretty well using all procedures; GLS is particularly adapted to them as well as AGLS.

If the best fit is the selection criterion, there is no doubt that partly rescaled data with polyserial and polychoric transformation and completely rescaled ordinal data with polychoric transformation should be the most suitable methods (methods 3 and 5 for the Robust method and 3 for the AGLS method).

The Asymptotic Generalized Least Squares method is a GLS method that can have arbitrary distribution acceptable in large samples. Empirical studies tend to show that the AGLS method could break in small samples. To correct this problem there are two new tests substantially more reliable: the Yuan-Bentler (1997) corrected AGLS statistics and the Yuan-Bentler F-test (1999). Both tests substantially correct the problem with the Chi-square statistic.

We can easily observe that the Chi-square statistic has substantially increased, the statistics are now more significant.

The best treatments of all should be the partial rescaling continuous data and the transforming of the Pearson correlation matrix with a polyserial and polychoric partition (option 3) using the Robust procedure of optimization first, and the completely rescaling continuous data to ordinal data with polychoric transformation (option 5) using the Robust procedure.

If we choose to use the Yuan-Bentler statistics with

small samples, the option 3 (the partly rescaled continuous data with polyserial and polychoric transformation with small samples) gives a better solution than option 5 (which is very acceptable too).

The second best treatment should be to rescale the data from a continuous scale to an ordinal scale: in this case the most significant procedures of optimization are GLS and AGLS. Rescaling the data completely eliminates those non-normal data, so the three optimization procedures fit the data very well (option 4).

Another very serious option to take in account should be, to completely rescale continuous data to ordinal data

and transform the Pearson correlation matrix into a polychoric matrix; when the data are still non-normal, this option should be used with the Robust optimization method (which gives a perfect fit). Another option should be to use the Yuan-Bentler statistics tests for smaller samples. In our example the Yuan-Bentler corrected statistics for option 5 are very close to those in option 4 (option 5). So option 4 is still an interesting option for the ML, GLS and AGLS optimization procedures.

The chosen data of our research are non-normal but the three optimization methods fit the continuous data pretty well (option 1). In the case of partial rescaling the continuous data (option 2), the fit increases for the AGLS optimization method, all other optimization methods staying the same. The fit for the AGLS optimization method will improve dramatically when rescaling and transforming the Pearson correlation matrix into a polyserial and polychoric matrix (option 3).

Complete rescaling the continuous data to ordinal data without any transformation of the Pearson correlation matrix, permits to fit the data very well for the three classic optimization methods (ML, GLS and AGLS).

All transformations of the Pearson correlation matrix (for partly or complete rescaled continuous data to ordinal data) should be used with the Robust procedure, to obtain acceptable good fits (options 3 and 5).

Conclusion

In Structural Equation Modeling the classical elimination of variables of the respecification process could suppress those non-normal variables, so that the remaining variables should certainly present a better distribution, and as the matter of fact the model could be better adjusted.

This is the case with our database, where the (observed) variable b4 could be eliminated (through the process using Modification Indices) and the fit will be improved from a 0.256 Chi-square probability to a 0.471 probability. This non-normal variable has been eliminated and the fit improved.

Once there is evidence of non-normality among continuous data, it is suggested, first, to try fitting data with the classic optimization methods like ML and GLS. However if the data are still not well adjusted to the model, we suggest using the Asymptotically Distribution Free estimation method (AGLS).

In case of necessity, we can also perform a bootstrap to identify the appropriate optimization method that should be more suitable to adjust the data.

In the event that data are severely non-normal, a rescaling transformation treatment with the purpose to reduce variability should be welcome. This implies to completely redefine the scale. If data are still non-normal and the fit with ML and GLS methods are bad, we will suggest to transform the Pearson correlation matrix into a polyserial and a polychoric matrix and to submit it again to the Robust and the AGLS optimization methods.

In the event that a polyserial or a polychoric partition of the correlation matrix has been performed and that the sample size is small (inferior to 100 observations or much less than 10 observations by variable), it is suggested analyzing data with the Yuan-Bentler corrected statistics tests (more suitable for smaller samples).

In the case of non-normal ordinal data it is suggested using, the Asymptotically Distribution Free optimization method (AGLS) first, this can correct the situation. If not, you should consider the transformation of the Pearson correlation matrix into a polyserial and /or polychoric matrix and then apply the Robust and /or AGLS optimization methods.

APPENDIX

Brief description of variables used in this research (see Lévy Mangin, 1997, 1999a and 1999b).

- b2: Customer satisfaction with products and brands.
- b3: Retailer satisfaction related to the provider influence in relation with the inventory.
- c1: Retailer satisfaction with the payment system.
- d3: Retailer satisfaction on purchases related to the leading provider.
- d6: Satisfaction in relation with the total discount received from the leading provider.
- e1: Global satisfaction with the leading pro-
- f1: Retailer satisfaction with its business profitability.
- f3: Retailer Satisfaction with its return on assets.

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