CONDITIONAL OR RELATIVE PROBABILITY

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"God doesn't play dice with the world"

Albert Einstein

1. Introduction

All objects exist in some environment or other. The scientist attempts to model material objects in terms of properties. All the properties of a material object-in-its-environment are lawfully related either deterministically (as in classical mechanics) or stochastically (as in quantum mechanics).

Since the probability calculus is used in physical science, biology, and sociology, in this paper we shall sketch the calculus of probability, CP for short, as a mathematical abstract theory. We shall present it in an axiomatic way following Rényi (1955, 1970a,b). Also we shall deal with some interpretations of CP, in particular its factual one as propensity of states or events.

2. Kolmogoroff's Axiomatization of CP

In CP we deal with the notion of 'event' and 'probability measure', as in the statement, "The probability of event x equals y". In principle any set qualifies as an 'event'. The probability of such an 'event' is a real number assigned to it by the probability function. The probability function P is defined on a family F of sets. (F must be a σ algebra, i.e. the union and the intersection of any two members of F are in F; also F is closed under complementation.)

Let then F be a σ algebra on a non-empty set S, and $P : F \rightarrow [0,1]$ a non-negative real-valued bounded function on F.

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Definition 1. *P* is called the *absolute* probability measure on *F* iff

(i) $P(R) \geqq 0$, for every $A \in \wp(S)$,

(ii) $P(A \cup B) = P(A) + P(B)$, for $A,B \in \wp(S)$ if $A \cap B \neq \emptyset$,

(iii) $P(S) = 1$, i.e., *P* is normed. ('e' abbreviates 'is a member of' and '$\wp(S)$' abbreviates 'power set of *S*'; in the following theorem '$\bar{A}$' denotes the complement of *A* in *S*= S - A.)

Theorem 1.

$P(\bar{A}) = 1 - P(A)$

Proof. From condition (ii), $\bar{A} \cup A = S$, and condition (iii) directly.

Theorem 2.

$0 \leqq P(A) \leqq 1$

Theorem 3.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof. (See figure)

$P(A \cup B) = P[A \cup [(A \cap B) \cup (\bar{A} \cap B)]] =$

$P[A \cup (A \cap B) \cup (\bar{A} \cap B)] = P[A \cup (A \cap B)] =$

$P(A) + P(\bar{A} \cap B)$ (1)

$P(B) = P[(A \cap B) \cup (\bar{A} \cap B)] = P(A \cap B + P(A \cap B)$ (2)

Subtracting (2) from (1)

$P(A \cup B) - P(B) = P(A) - P(A \cap B)$ Q.E.D.

(More theorems can be carried out. See Kolmogoroff 1950.)

Definition 2. A conditional probability measure, in Kolmogoroff's sense, is defined as follows:

$$P(A / B) =_{def.} P(A \cap B) / P(B), \text{ where } P(B) > 0 \tag{3}$$

Remark. If $P(A / B) = P(A)$ we say that A is independent of B. If A is independent of B, then B is also independent of A. Therefore $P(B / A) = P(B)$.

In this case, from (3) it follows that $P(A \cap B) = P(A) \cdot P(B)$.

3. Rényi's Axiomatization of CP

The probability function $P(A / B)$ maps pairs A, B of sets of a family F of sets.

Let F be a σ algebra on a non-empty set S, and $P(A / B) : F \to [0, 1]$ a non-negative real valued function on F (i.e., the power set of S).

Definition 3. $P(A / B)$ is called the conditional probability measure of the event A with respect to the event B, where $A \in \wp(s)$, and $B \in \beta \subseteq \wp(S)$ (where $\wp(S)$ stands for the power set of S) iff it satisfies the following axioms:

Axiom 1. $P(A / B) \geqq 0$, further $P(B / B) = 1$ for every $A \in \wp(S)$ and $B \in \beta$.

Axiom 2. $P(A_1 \cup A_2 / B) = P(A_1 / B) + P(A_2 / B)$ for any $B \in \beta$, provided

(a) $A_1 \cap A_2 = \emptyset$; and

(b) $A_1, A_2 \in \wp(S)$.

Axiom 3. For any fixed $B \in \beta$, if $A_1, A_2 \in \wp(S)$ and $A_2 \cap B \in \beta$, $P(A_1 / A_2 \cap B) \cdot P(A_2 / B) = P(A_1 \cap A_2 / B)$.

In what follows, if $P(A / B)$ occurs, it is tacitly assumed that $A \in \wp(S)$ and $B \in \beta \subseteq \wp(S)$.

<u>Theorem 1.</u>

$P(A / B) = P(A \cap B / B)$.

<u>Proof.</u> If in Axiom 3, $B = A_2$, we have $P(A_1 / B).P(B / B) = P(A_1 \cap B / B)$. Taking into account Axiom 1, namely $P(B / B) = 1$, Theorem 1 follows.

<u>Remark</u>. $P(S / B) = 1$

By Theorem 1 we have $P(S / B) = P(S \cap B / B) = P(B / B)$; by Axiom 1, we get $P(S / B) = 1$.

<u>Theorem 2.</u>

$P(A / B) \leqq 1$.

<u>Proof.</u> According to Axiom 2, we have

$P(A \cap B / B) + P(\bar{A} \cap B / B) = P(B / B)$. By Axiom 1,

$P(B / B) = 1$, and $P(\bar{A} \cap B / B) > 0$. Therefore, it follows

$P(A \cap B / B) \quad 1$, and by Theorem 1, $P(A / B) \leqq 1$.

<u>Theorem 3.</u>

$P(\emptyset / B) = 0$.

<u>Proof.</u> According to Axiom 2,

$P(\emptyset / B) = P(\emptyset \cup \emptyset / B) = 2P(\emptyset / B)$. Therefore,

$P(\emptyset / B) = 0$.

<u>Remarks.</u>

1. It follows from Theorem 3 that $\emptyset \notin \beta$, because if $\emptyset$ belonged to β, we would have $P(\emptyset / \emptyset) = 0$ by Theorem 3. Thus the assumption $\emptyset \in \beta$ leads to contradiction.

2. If $A \cap B = \emptyset$, then from Theorems 1 and 3, $P(A / B) = 0$.

<u>Theorem 4.</u>

If $A \subseteq A' \subseteq B' \subseteq B$, then $P(A / B) \leq P(A' / B')$.

<u>Proof.</u>

$P(A / B) = P(A \cap A' \cap B' / B) =$

$P(A \cap A' / B' \cap B).P(B' / B) \leq P(A \cap A' / B') =$

$P(A' / B') - P(\bar{A} \cap A' / B') \leq P(A' / B')$.

Q.E.D.

<u>Remark.</u> If $A = A'$, then $A \subseteq B' \subseteq B$. Therefore,

$P(A / B) \leq P(A / B')$.

For more theorems see Rényi 1955, 1970 a,b).

The advantage of Rényi's approach is that his axiomatization contains Kolmogoroff's as a special case. It is richer than Kolmogoroff's as we shall see presently.

4. <u>From Rényi's to Kolmogoroff's Axiomatization of CP</u>

If <u>P</u> is a probability measure defined on a σ algebra <u>F</u> of subsets of the set <u>S</u>, and further if $P(S) = 1$, then the triple $<S, F, P>$ is called a <u>probability space</u> in the sense of Kolmogoroff.

If we define $\mathfrak{B}$ as the set of those sets <u>B</u> for which $P(B) > 0$, and put $P(A / B) = P(A \cap B)/P(B)$ for $A \in F$ and $B \in \mathfrak{B}$, then the quadruple $< S, F, \mathfrak{B}, P(A / B) >$ is a <u>conditional probability space</u> generated by the probability space $<S, F, P>$.

Conversely, if $<S, F, \beta, P(A / B)>$, where $\beta \subseteq \wp(s)$ (recall Section 3), is a conditional probability space and <u>C</u> is an arbitrary element of β, putting $P_c(A) = P(A / C)$, then $<S, F, P_c>$ will be a probability space in the sense of Kolmogoroff.

Let us see the connection between Rényi's and Kolmogoroff's approach. If we put $P_c(A) = P(A / C)$ for $A \in F$ with $C \in \beta$ fixed, and define, as usual in the theory of Kolmogoroff, the conditional probability $P^*(A / B)$ for a set $B \in \beta$ for which $P_c(B) > 0$ by $P^*(A / B) = P_c(A \cap B) / P_c(B)$, then we have

$P^*(A / B) = P(A \cap B / C) / P(B / C)$ and by Axiom 3

$P^*(A / B) = P(A / B \cap C).P(B / C) / P(B / C) = P(A / B \cap C)$.

Remarks.

1. In case $S \in \beta$, clearly $<S, F, P_s>$ is a probability space in the sense of Kolmogoroff.

2. In the previous case, remark 1, $<S, F, \beta, P(A / B)>$ may not be identical with the probability space generated by $<S, F, P_s>$ because may contain sets $\underline{B}$ for which $P(B / S) = 0$ and at the same time need not contain every set $\underline{B}$ for which $P(B / S) > 0$, i.e., the system consisting of all sets $B \in F$ for which $P(B / S) > 0$ need not be identical with β.

3. If $P^*(A / B)$ is defined by $P^*(A / B) = P(A \cap B / S) / P(B / S)$ for $B \in \beta_s$, we have $P^*(A / B) = P(A / B)$, provided that $B \in \beta$.

Rényi's approach then is richer than Kolmogoroff's and contains the theory of Kolmogoroff as a special case.

5. Some Interpretations of CP

We said above that CP is an abstract mathematical theory devoid of any specifical factual content. Now we can interpret the members of the universe $\underline{S}$ as a collection of events (propensity interpretation), data (frequency interpretation), statements (some logicians claim that $\underline{S}$ could only be a set of propositions (1) and P(A) the probability that $\underline{A}$ be true (2)), or beliefs (for criticism of the subjectivistic interpretation of probability as the degree of belief see Popper 1956-1982).

Here is a sample of interpretations:

(i) Propensity interpretation (objective probability): interpretation of $\underline{S}$ = set of possible physical events of a kind; interpretation of $P(A / B)$ =natural dispositions of events(s) $\underline{A}$ to happen in environment $\underline{B}$;

(ii) Randomness interpretation (objective chance): Interpretation of $\underline{S}$ = set of random (mutually independent) events of a kind; interpretation of P(A / B) = the objective odds of event(s) A given an environment B;

(iii) Statistical interpretation (relative observed frequency): interpretation of S = population of empirical data of a kind; interpretation of P(A / B) = relative frequency of the sample A of observations in the context B. (A has to be a random sample of S. P(A / B) are rational numbers included in the unit interval. Empirically found frequencies are therefore bound to differ from calculated probabilities: while probability values are real numbers, frequencies are rational.)

6. Propensity in Quantum Theory

That chance is objective has been stressed, among others, by Popper (1957, 1959). While some authors interpret P(A) as the natural disposition of event(s) A to happen, i.e., propensities are interpreted within the framework of "absolute" probabilities, Popper's interpretation applies to a quantum-system within an environment. (See Settle 1974 for different interpretations of probability in quantum theory.)

The probability of an event in some environment suggests not separating the two, but regarding them as different features of a single indivisible block: event-plus-surrounding. The statement 'P(A / B) = r $\in$ [0, 1]' can be interpreted in two ways:

(i) the system "has" the probability;

(ii) system and environment enter the situation on an equal basis, the probability belongs to both.

In our interpretation, in the above statement r is predicated of a relation in which the quantum-system stands to the environment, and not to the quantum-system taken by itself. As Cini (1983, p. 53) reminds us:

> "I want physics to describe reality without any reference to the community of observers. But what I mean by this is that it is not possible to describe reality without any reference to the collection of instruments and artifacts placed by the experimenter in order to detect the properties of the objects he is interested in. Not absolute properties, therefore, but properties which depend on the conditions he has chosen to operate with. I do not want to speak of observers because they do not have any role in the definition of the physical reality. They do not interfere with reality when they look at a dial or a pointer".

A similar interpretation is supported by Fock (1971, p. 301):

"Probability is to be regarded as an essential element of the description and not as an indication of the incompleteness of our knowledge [as Einstein thought]; this follows already from the fact that, for given external conditions, the result of the interaction of the object with the measuring instrument is (in the general case) not predetermined, but has only some probability. A series of interactions leads statistics that correspond to a definite probability distribution. This probability distribution reflects the potentiality that existed in the given conditions before the interaction had taken place."

And by Omelyanovsky (1979, p. 136) when he states that in quantum mechanics, "probablities are considered as ocurring in the basic laws of nature, and their introduction reflects the potentially possible objectively existing in certain real conditions".

Probability then is no longer interpreted as the measure of human knowledge or ignorance but it is at the very basis of the laws of quantum mechanics. Further, probability underlines the main trends in modern scientific research, from physics to biology to sociology.

I claim, following Einstein, that the world of photons, electrons, and the like exists <u>out there</u>, even if we do not observe it. Further, we must accept the qualitative difference between the quantum world and the classical world, <u>pace</u> Einstein himself (Tobar-Arbulu 1985a). According to quantum theory some properties, i.e., the position, momentum, spin of a quantum-system (electron, proton, photon or whatnot) are represented by random variables. Every property is representable by a variable which in turn is an operator. The variables then are assigned probabilities. Therefore, in quantum theory probability is primary and irreducible (3). Further, relative or conditional probability is the mathematical tool (Rényi 1955, pp. 285-6) which stands for the objective propensity of the physical object-environment complex. Quantum physics introduces a random variable $\Psi^* Q \Psi$, where Q is an operator acting on Ψ, a place and time dependent function that determines the position probability density $\rho = |\Psi(x)|^2$. We consider, then, propensities as physical and real. They are time dependent. Further, Schrödinger equation tells us how probabilities change as time goes by. Schrödinger equation is a law of changes of probabilities along time.

$\Psi^* Q \Psi$ are possesed all the time by the quantum-system, they are real (though not directly observable) properties.

Every one of the random variables stands for a property of an individual thing-in-its-environment, not a collection property of a whole bunch of similar entities. Thus some stochastic properties (represented by random variables) are basic. Further, since the physical object in question is always in some environment or other (there is no thing without environment except the universe as a whole), all the properties are properties of a physical object-in-its-environment. (Needless to say, here we are dealing with an objective interpretation of quantum theory free from measuring instruments, let alone from observers.)

To repeat, propensities are physical and real (Popper 1956-1982). Moreover, in quantum theory we deal with superposition of probability amplitudes and their interference. (For example, the initial superposition state $\Psi = c_1\Psi_1 + c_2\Psi_2$ "collapses" into state Ψ_1 with probability $|c_1|^2$ or into state Ψ_2 with probability $|c_2|^2$.) The superposition principle, which is responsible for adding amplitudes instead of probabilities, is one of the laws of nature that we must accept as we accept the universal constancy of the speed of light. This principle is responsible for the peculiar interference phenomena that lead to quantum correlations quite different from classical ones. Physical reality at the quantum level connot be defined in classical terms, as was attempted by Einstein, Podolsky and Rosen in their famous paper. The superposition principle is a principle of nature and applies to single (though complex) individuals, such as molecules.

So every individual is normally in a superposition of eigenstates. Thus we can say that quantum theory deals with <u>potentialities</u> (4) that get actualized only exceptionally, namely during measurements. The measuring apparatus filters out all eigenstates "projecting" the one corresponding to the eigenvalue it measures.

As for measurement, with Margenau (1958), Pauli (1980, p. 75) and Fock (1982, p. 19), we distinguish the act of measurement from the preparation of the quantum-system. In a specific measurement the potentialities quantum theory is about get actualized (Fock 1982, p. 20): In this case the environment is the apparatus itself. Thus, Cini (1983) has developed a realistic theory of measurement, treating the measurement process as a particular one ocurring in a physical system. He has studied the specific measurement of a single individual with polarized counters and Stern-Gerlach devices as a micro-macro interaction. Cini has shown that the so-called "collapse of the wave-packet" or "projection of the state function" occurs with a high degree of accuracy, though not instantaneously, and as a consequence of basic physical laws (5). The change brought about in the physical system due to the interaction with the measuring apparatus is not interpreted to mean that "reality is created by measurement". On the contrary, "as soon as one takes into account the physical fact that only the microsystem together with the apparatus can be correctly represented as an isolated system, the objectivity of reality is restored and the causal evolution of the state vector [gives] information about the possible different outcomes of the interaction of the microsystem with the apparatus" (Cini 1983, p. 50).

7. Conclusions

Our consideration of propensity in quantum theory is free from any empiricist traits. However, it does take into account the physical environment. Further, the act of measurement takes into account the apparatus as a specific environment. Our interpretation is close to Fock's. In fact, Omelyanovsky, after reminding us (1983, p. 51) that "the very nature of the micro-object is the basis of the fact that its description is unthinkable without probabilistic concepts and potential probability [...] for probability itself is part of the laws of quantum mechanics", states (Omelyanovsky 1983, p. 50), "in describing phenomena on the atomic scale one must not ignore the physical conditions (experimentally recorded) under which these phenomena are observed. This kind of relativity with regard to the experimental devices or instruments of observation (the concept and term "relativity with regard to the instruments of observation" were first introduced by Fock) [...] is a distinctive feature of the description in quantum mechanics". (For the notion of relativity in quantum mechanics with regard to the means of observation see Fock 1971, 1982.)

Thus with Fock (1971, p. 300) we can say that in quantum mechanics "we introduce a new, important notion of relativity with respect to means of observation which constitutes a generalization of the old and well-known notion of relativity with respect to a reference frame". I should add, however, that this new principle of relativity is with regard to the surrounding physical environment and it requires that physical reality has to be considered as given in a physical reference frame.

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NOTES

(1) The claim (Carnap 1950) that probability is a certain logical relation between propositions is wrong. In fact, there are no objective procedures for assigning probabilities to propositions, in particular to probabilistic hypotheses such as probability distributions. On the contrary, such assignments are made either as the strenght of measurements (as in technology) or of hypothetical random mechanisms (as with genetic mutation in biology). The "probability of a proposition" connot be interpreted as "the probability of the fact referred to by the proposition".

(2) The notion of probability has been used to elucidate the concept of partial truth. This approach is misleading because the notion of partial truth is prior to the concept of probabilty statement (Tobar-Arbulu 1986b). Further, we must have an independent notion of truth if we want to know what the value of a given probability statement is.

(3) In Holton's words (1984) nowadays physical themata presuppose fundamental probabilism. I.e., probability in modern physical theories is primary and irreducible. It is to be regarded as "an essential element of the description and not as an indication of the incompleteness of our knowledge" (Fock 1971, p. 301). Similar considerations can be made in other fields: "The problem of the relation between a living system and the surrounding medium bears some ressemblance with that of the relation between an atomic object and a measuring device: an organism is characterized by its own properties that are not reducible to those of the medium, but it cannot exist independently and irrespectively of the surrounding medium" (Fock 1971, p. 3055).

(4) According to Pauli (1946, p. 30), "I shall only recall that the statements of quantum mechanics are dealing only with possibilities, not with actualities [...] The actual observation appears as an event outside the range of a description by physical laws and brings forth in general a discontonuous selection out of several possibilities foreseen by the statistical laws of the new theory". (See, however, Cini 1983 for a realistic interpretation of the act of measurement.) Dirac (1928) at the fifth Conseil Solvay of Physics on "Electrons and Photons" in Brussels, after commenting on the essential differences between classical and quantum description of physical processes dealt with the so-called 'wave-function'. Quantum theory, according to him, describes a state by a time-dependent wave-function Ψ, which can be

expanded at a given time t in a series containing wave functions Ψ_n with coefficients c_n. The wave functions Ψ_n are such that they do not interfere at an instant $t > t_1$. For Dirac, Nature makes a choice some time later and decides in favor of that state Ψ_n with probability $|c|^2$. This choice cannot be renounced and determines the future evolution of the state. What Dirac calls "choice of Nature", we have called "objective propensity" (Tobar—Arbulu 1986a). The quantum mechanical principle of superposition of states is a principle of Nature and has to be interpreted only in this way. Thus the quantum object is characterized by a definite set of possibilities. In the interaction of this quantum object with the apparatus (Cini 1983) the possible gets actualized.

(5) Cini (1983) has proved quite convincingly that the postulate of wave packet collapse, introduced as an extra assumption in quantum mechanics in order to describe the change in the wave function of a quantum object ocurring during the time interval which is necessary to perform the measurement of one of its physical variables, can be dropped and replaced by the Schrödinger time evolution of the state vector of the total system object plus apparatus. This is proved, strictly speaking, only for the ideal measuring processes described by the interacting Hamiltomians introduced in his paper, and not for any conceivable measuring apparatus. But, within these limits, the elimination of the projection postulate from the conceptual foundations of quantum mechanics is shown to be consistent with the known features of any physical measurement.

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RESUMEN

En el trabajo se presenta el cálculo de probabilidades como una teoría matemática abstracta. Para ello seguimos los enfoques de Kolmogoroff y Rényi. Se prueba que el enfoque de Rényi es más rico que el de Kolmogoroff, pues incluye a éste.

Dado el carácter abstracto del cálculo de probabilidades, puede ser interpretado de forma diferente: como propensidad, azar o cambio objetivo y, estadísticamente, como frecuencia observada.

En cuanto a la interpretación como propensidad, se da una interpretación de la mecánica cuántica no-relativista siguiendo a Bohr. Es decir, las probabilidades que aparecen en esta teoría física se interpetan como irreducibles (por tanto, no expresan ninguna subjetividad del observador) que hacen referencia a la potencialidad o propensidad que posee un sistema cuántico en un cierto medio ambiente. La probabilidad se refiere, por tanto, a relaciones, como una y otra vez ha repetido Bohr. Tales potencialidades únicamente se actualizan en determinadas ocasiones: una de ellas cuando se realiza la medición.