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Trabajo de Fin de Grado

Handling of Race Cars

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RESUMEN:

El siguiente proyecto aborda de forma detallada la dinámica vehicular de coches de competición y cómo afecta esta a la maniobrabilidad. La dinámica es uno de los aspectos más importantes a tener en cuenta para el diseño de vehículos de producción, y más aún, debido a la naturaleza competitiva y el alto ritmo de evolución, en los vehículos de competición. Se comienza desarrollando un estudio sobre el comportamiento de los neumáticos, analizando las fuerzas que actúan sobre ellos y las distintas variables a tener en cuenta. A continuación, nos encontramos con un capítulo en el que se explica la propia dinámica del vehículo al completo, analizando las respuestas del vehículo en estado estacionario y en estado transitorio. El siguiente capítulo trata sobre el frenado, en el que se indagan las fórmulas fundamentales para su entendimiento y algunos de los sistemas que se utilizan en la alta competición. Siguiendo con la temática, el apartado cinco es un estudio sobre la dirección y las geometrías utilizadas para optimizar el rendimiento. Para finalizar, se estudian las técnicas de conducción y los posibles reglajes que ayudan a bajar el tiempo por vuelta.

ABSTRACT: The following project deals in detail with the vehicle dynamics of racing cars and how it affects handling. Dynamics is one of the most important aspects to be taken into account in the design of production vehicles, and even more so, due to the competitive nature and the high rate of evolution, in racing vehicles. We begin by developing a study on the behaviour of tyres, analysing the forces acting on them and the different variables to be taken into account. This is followed by a chapter in which the dynamics of the vehicle itself is explained in full, analysing the responses of the vehicle in steady and transient states. The next chapter deals with braking, in which the fundamental formulas for its understanding and some of the systems used in high competition are investigated. Continuing the theme, section five is a study of steering and the geometries used to optimise performance. Finally, driving techniques and possible settings to help lower lap times are examined.

Palabras clave: Vehicle dynamics, handling.

Firma del alumno:	VºBº Tutor/es:
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*“Ve con confianza en la dirección de sus sueños. Vive la vida que has imaginado.”
Henry David Thoreau*

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1. Introduction

1.1. Basic concepts

It is important to start defining some fundamental definitions that will appear in the following chapters. As this thesis is about vehicle dynamics and handling of race cars, could be a good idea to start defining what is a vehicle.

There are two types of ground vehicles: those using tracks on the ground to move in a fixed direction (such as railway vehicles) and those not using tracks and that can move in any direction by steering the wheels (such as road vehicles). These last type of vehicles are capable of independent motion on the ground using the forces generated by its own motion. Vehicles that will be treated in this text belongs to the second category. [1]

Another important definition is the tyres, that constitute the only contact between a car and the track, and thus deserve the most attention and care of any race car. The design of the car must be optimized to optimize its tyre performance. Tires have to withstand different loads during a race. They must first offer grip, which then has to extend to acceleration, braking and cornering forces at the limit. They are expected to do two things at once: braking and cornering and accelerating and cornering. They also need to provide feedback so the driver can get the tires up to their maximum level of grip. [2]

Tyre characteristics are determined by their physical structure and its compound. The compound will define the life of the tyre and the level of grip.

Cross-ply tyres have plies that cross over one another at a 45° angle from the tyre's rotation, which tends to result in less static camber since plies are shared with the sidewalls and tread, making them distort together. As a result of their elastic flex, cross-ply tires usually offer more feedback and are more progressive, in terms of grip levels, than other tyre types; they are therefore able to handle a wider range of slip angles. [2]

Radial-ply tyres are more popular and have plies that run at a 90-degree angle from the tyre's rotation. On account of more plies running around the tread, the sidewall flexes, driving the need for more static camber. They often provide more peak grip but are more likely to break away with less feedback.[2]

In dry conditions, teams choose slick tyres, which do not contain grooves or cuts in the tread, and this drive to the maximum contact patch, and therefore, grip.

On the other hand, wet tyres have a large amount of cuts and grooves to help channel the water away from the contact patch to avoid aquaplaning.

Wet tyres typically use a softer compound because the water cools the tyre quickly, requiring a softer compound to maintain the optimum working temperature [2].

In vehicle dynamics is also important to understand what is the sprung and unsprung mass. The weight of the vehicle supported by its suspension springs, such as the chassis, engine, driver, and gearbox, is referred to as sprung mass. The heavier the sprung mass, the stiffer the spring must be to keep the car off the ground. The weight that is not supported by the springs, such as the wheels, tyres, brakes, and uprights, is referred to as unsprung mass. Because this is uncontrolled, the lighter it is, the better the contact between the tyre and the road surface, especially during transitions and when tackling bumps and undulations [2].

The vehicle is directly influenced by the forces exerted by the road surface, the atmosphere and gravity. Therefore, we should be interested in the aerodynamic properties of the car and in the characteristics of the tyres and the forces exerted on them. For most of the practical cases, tyre forces are the predominant forces to be taken into account, so aerodynamic aspects will not be dealt with in this text. The aim of this project is to provide information on the most important parameters influencing the behaviour of the vehicle. The effects of the different suspension configurations have not been assessed either. Chapters on aerodynamics and suspension would make the text too dense.

1.2. What is Vehicle Dynamics

According to the definition given by William Milliken, "Vehicle dynamics, as we use

the term, is the branch of engineering which relates tire and aerodynamic forces to overall vehicle accelerations, velocities and motions, using Newtons Laws of Motion. It encompasses the behavior of the vehicle as affected by driveline, tires, aerodynamics and chassis characteristics. The subject is a complex one because of the large number of variables involved. "[3]

Vehicle Dynamics is divided into two major categories: isolation and control.

Isolation is all about isolating the driver from external disturbances caused by vehicle operation and is divided into two topics as well. The first category includes disturbances caused by the vehicle itself, such as engine vibrations. The term "refinement" can be applied to this category. The second category includes disturbances caused by the outside world, such as aerodynamic interactions between the vehicle and its surroundings (wakes from other cars or crosswinds) and road imperfections. The reaction to these imperfections is known as "ride." [4]

There is an important crossover in aerodynamic behaviour between control and isolation because control implies the rejection of disturbances ("fidelity") and the absence of their amplification ("stability"). Something similar occurs when a response to road disturbances is a change in the vertical load supported by the tire, which has a strong influence on the lateral force that the tire is generating at any given instant in time, being crucial for fidelity and stability. So we can make a reflection that one of the difficulties of vehicle dynamics is not the complexity of the individual effects, but rather the complexity of their interactions.

Control refers to the vehicle's behavior in response to driver actions.

The speed variation depends on the mass of the vehicle and the available traction. Most cars have a steering wheel that requires a "yaw rate" and a combination of a forward velocity vector rotating with the vehicle and a yaw rate results in a curved path. The availability of the maximum track curvature (in normal driving), i.e. the curve radius, is only available at the lowest speeds.

Another thing that is important to know is that it is generally true that the vehicle does not behave geometrically and its turning radius cannot be predicted by considering the angle of the front wheels in relation to the rear wheels, except under about 48 km/h (13.33 m/s). [4]

In everyday road use, the driver moves the steering wheel slowly, allowing the vehicle to easily respond to the required yaw rate as it is within performance limits.

However, if the driver increases the yaw rate demand amount, the vehicle increases its yaw rate proportionally, as shown in Figure 1.1. This property is called "linearity" and for the driver the behavior of the vehicle is instinctive. [4]

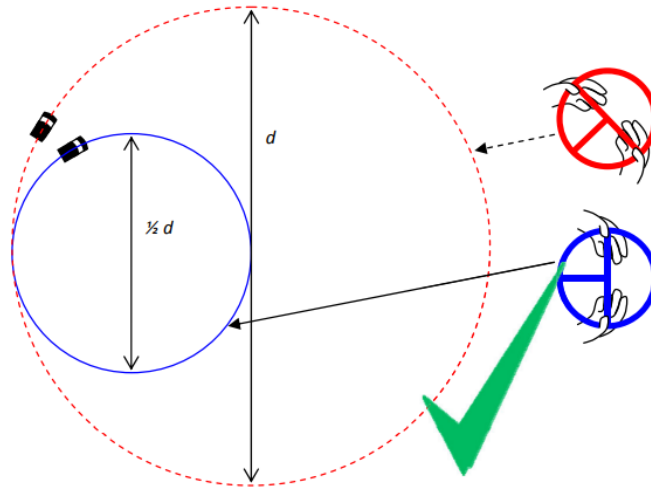


Figure 1.1. Linearity, more input on the steering wheel results in a proportionally more yaw rate. [4]

The restriction for lateral acceleration and yaw rate is imposed by the friction available between the tyres and the track. Over a dry asphalt, the tyres of a regular road vehicle have a coefficient of friction of approximately 0.8-1. This is to say, tyres can develop a force, both lateral, longitudinal or mixed of among eighty to 100% of the load that falls over them. In this cars, restriction accelerations not often exceed 9 m/s^2 , with yaw rates at around 12 deg/s at speeds of approximately 40 m/s . However, In competition cars, the values of the coefficient of friction and forces developed by the tyres are the double, with pressures much more lower (1.2 bar). For competition vehicles, in addition with large aerodynamic downforces, allow lateral acceleration in the region of 30 m/s^2 with yaw rates correspondingly over 40 deg/s for a speed of 40 m/s . [4]

The vehicle becomes strongly "non linear" when racing or in emergency situations attempting to avoid an accident for example. The steering wheel is moved rapidly and the vehicle usually has difficulties in responding to the movements in the steering wheel. This area is named as "Transient Handling" and is an important object of studies during the design process. In transient behaviour, all the vehicle states fluctuate rapidly, and is connected with steady state.[4]

Transient driving behavior studies focus on the analysis and understanding of the yaw moments acting on the vehicle and its response to them. These moments are

determined by the lateral and longitudinal forces of the tires. In road cars, aerodynamic forces are a small modifier, but for competition vehicles, these aerodynamic loads are really important. [4]

Tyres generate lateral forces by two mechanisms, "camber" and "slip angle". Camber is the deviation of the tyres in relation to the vertical axis when viewed from the front. In the rear tyres is desirable to have the largest surface area in contact with the road, so that camber is close to 0. On the other hand, negative camber on the front tyres helps to put more surface area in contact when cornering. Slip angle is the difference between a rolling wheel's actual direction of travel and the direction towards which it is pointing. At speeds under about 16 km/h (4.4 m/s) the lateral stiffness of the tyres generate the forces that constrain the vehicle to its intended path. Slip angle forces can be more than 20 times camber forces for a particular angle, so it is the most important aspect for vehicle dynamics.[4] Lateral forces induced by the angles are strongly modified by the vertical load on the tyres at each moment in time.

Tires generate longitudinal forces by rotating at a speed other than their "free-rolling" speed, which is the speed at which the wheel and tire would rotate if no braking or traction forces were applied to them. The difference in speed is referred to as the "slip ratio". It is expressed as a percentage. For example, a tire turning with a 5% slip ratio will take 105 revolutions to travel the same distance as a free-rolling tire making 100 revolutions [4].

The suspension linkage must handle the lateral forces on the tires by controlling slip and camber angle. With the front wheels, the driver can vary the slip angle using the steering wheel. Coping with the vertical loading of the tires is a function of the "calibration" of the suspension (springs, shock absorbers, etc.). The vehicle's braking and transmission systems have to cope with longitudinal forces. The modelling of tire forces and moments at the tire to road contact patch is one of the most complex issues in vehicle handling simulation [4].

Camber forces can be neglected from the behavior of tires for vehicles that do not roll freely. The slip angle is the dominant mechanism for generating lateral forces. A tire adjusts its slip angle to withstand the required lateral force. All tires have a slip angle at which maximum lateral force is generated, often referred to as the "critical slip angle". If the force required is greater than the force generated at the

critical slip angle, the tire will run up to and beyond the critical slip angle. [4]. Back on the vehicle, the lateral force requirement for the rear tires increases while it is stable or decreasing for the front tires. The rear tires experience an increasing slip angle while the front tires experience a constant or decreased slip angle. At this point, if the rear tires exceed their critical slip angle, their ability to eliminate yaw moment is lost. The only way to eliminate the yaw moment is to reduce the forces on the front tires. If the yaw moment persists, then the yaw acceleration will also persist. As the yaw rate increases, the slip angle on the front axle decreases while it continues to increase on the rear axle, which means that the rear tires are no longer able to absorb the vehicle's yaw moment. Additionally, if the front tires exceed their critical slip angle, the normal stabilization mechanism is reversed, resulting in an accelerating spin that departs quickly from the driver's control [4]. The primary goal of vehicle dynamics is to keep vehicle behavior within limits that the driver can understand and control. The increasing use of electronic systems in vehicles is driving the application of standards for the safe development of such critical standards, the most common of which is the automotive ISO26262 standard [4].

1.3. Vehicle Basic Scheme

The main important point when defining a mathematical model of a vehicle is to state clearly the assumptions behind each simplification and making clear under which conditions the model will predict the behaviour of a real vehicle.

There are two types of assumptions, the first one involving operating conditions and the second one concerning the physical model of the vehicle [5].

Regarding the operating conditions, several options can be envisaged:

Performance: the vehicle goes straight on a flat road, possibly braking or accelerating.

Handling: the vehicle makes turns on a flat road, usually with an almost constant forward speed.

Ride: the vehicle goes straight on a bumpy road, with constant forward speed.

Real conditions are a mixture of all of them.

A physical model may have the following features:

- (1) The vehicle body is a single rigid body.

- (2) Each wheel hub is connected to the vehicle body by a single degree of freedom linkage (independent suspension).
- (3) The steering angle of each front wheel is mainly determined by the angular position δ of the steering wheel, as controlled by the driver.
- (4) The mass of the wheels (unsprung mass) is very small if compared to the mass of the vehicle body (sprung mass).
- (5) The wheels have pneumatic tyres.
- (6) There are springs and dampers between the vehicle body and the suspensions, and, likely, between the two suspensions of the same axle (anti-roll bar). Front to rear interconnected suspensions are possible, but unusual.
- (7) There may be aerodynamic devices that may significantly affect the downforce.

A vehicle basic scheme is shown in figure 1.2, which also serves the purpose of defining some geometrical parameters:

- (1) The vehicle longitudinal axis x , and hence the vehicle heading direction \mathbf{i} .
- (2) The height h from the road plane of the center of gravity \mathbf{G} of the whole vehicle.
- (3) The longitudinal distances a_1 and a_2 of \mathbf{G} from the front and rear axles, respectively.
- (4) The lateral position b of \mathbf{G} from the axis.
- (5) The wheelbase $l = a_1 + a_2$
- (6) The front and rear tracks t_1 and t_2 .
- (7) The geometry of the linkages of the front and rear suspensions.
- (8) The position of the steering axis for each wheel.

All distances mentioned before are positive, except b , which will be typically set equal to zero. During the vehicle motion, there are suspension deflections, and this geometrical parameters can suffer small changes, so it is a common practice to take their reference value under static conditions (The vehicle is moving straight on a float road at constant speed).[5]

The study of performance and handling of vehicles is simplified under the hypothesis of small suspension deflections, assuming very stiff springs (Often the case for race cars). [5]

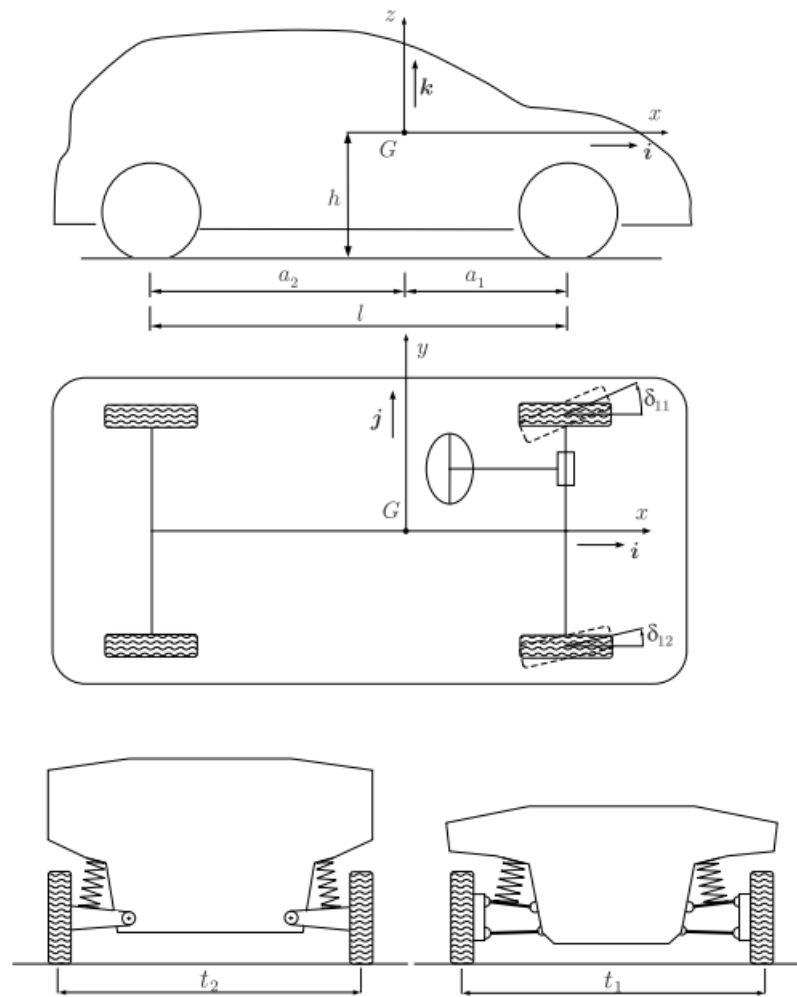


Figure 1.2. Vehicle Basic Scheme.[5]

1.4. Coordinate system

Two coordinate systems are frequently used. One called the Earth-fixed axis system and one for the vehicle in movement. In the fixed system capital letters X, Y and Z are used to denote the axis. X and Y are horizontal and at right angles to each other. Z is vertical downward [3]

The fixed system is used only when it is necessary to reference some aspect of the vehicle motion to a fixed point or direction.

The second coordinate system used is the vehicle axis system, commonly referred to as the "moving axis system" because it moves with the vehicle. The main important idea to remember is that it is fixed to the vehicle and the relative inertial

properties are assumed to be constant. As with the earth-fixed axis, the system is orthogonal and right-handed. The x-axis is horizontal and forward-facing when the vehicle is traveling straight on a flat road, the y-axis points to the driver's right, that is horizontal and at 90° to the x-axis. The z-axis is perpendicular to the other two, vertical and positive down [3].

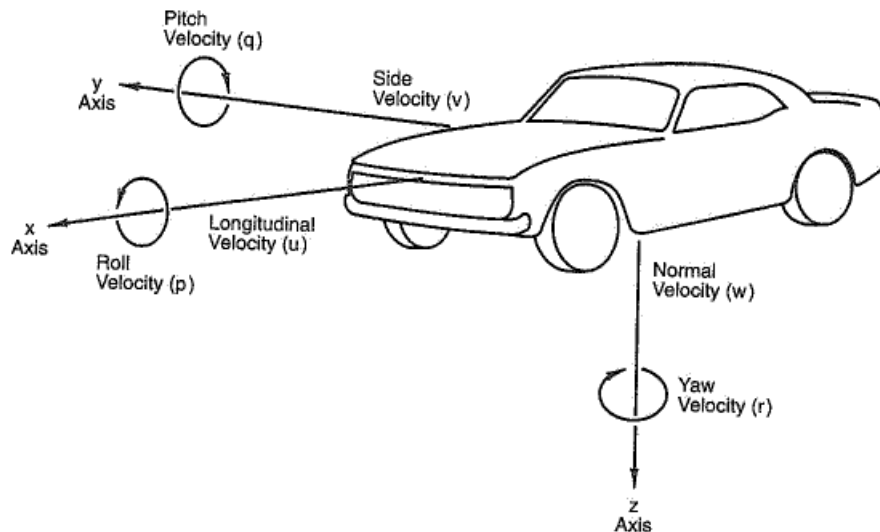


Figure 1.3. Vehicle axis system. [3]

1.5. A Vehicle Dynamics Overview

1.5.1. Racing

The main objective in racing is to obtain a setup capable of traversing a course in the shortest time or at the highest average speed when operated manually by a driver. A racetrack can be thought of as a series of segments, each consisting of a corner, a straight, and a corner. Figure 1.4 shows a plot of speed versus distance for eight of these segments. At point A, the car has reached the apex of a corner and is just beginning to accelerate on the straight; heavy braking at point B, consequently speed decreases as you approach curve C. The curvature of the frame near corners is due to braking and acceleration during cornering, and the decrease in acceleration at higher speeds is due to aerodynamic effects (drag) and the use of higher gear ratios [3].

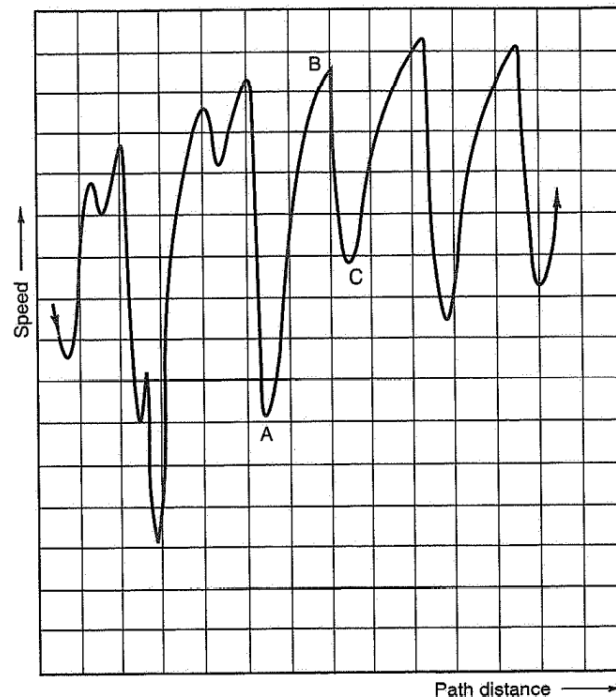


Figure 1.4. Circuit simulation results. [3]

An important aspect in circuit racing is that velocity should never be constant (as shown in figure 1.5) unless for reasons as traffic, safety, endurance, etc.

The driver should accelerate at the exit of each turn and continue to the point where, with a maximum braking (deceleration) the speed can just be brought down to the maximum speed for the next corner [3].

Using velocity vectors, the performance of a race car on a segment of a circuit can be expressed as in figure 1.5. The vector directions are tangent to the path of the center of gravity (CG) of the vehicle and the length of each vector is proportional to the speed at that point. Figure 1.5 indicates that velocity vector variations are product of changes in both speed and direction [3].

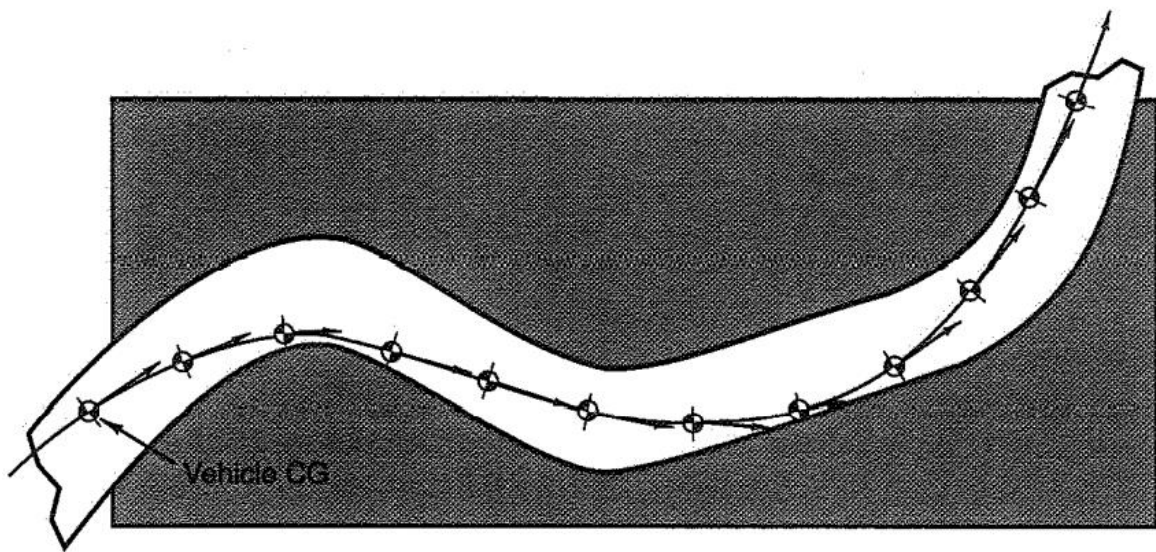


Figure 1.5. Vector velocity representation of race car performance. [3]

Race car requirements are best expressed in terms of acceleration because, as noted before, velocity vector is constantly changing, and according to the definition, acceleration is the rate of change of speed with time on a straight line, the acceleration can be also negative when braking, for example. The race driver is conscious of an additional form of acceleration, the cornering acceleration, this is associated with the change in direction of the velocity vector with time. [3]

For clarify these acceleration components, consider the following.

Figure 1.6(a) is a vector velocity diagram. The velocity vector V_1 is shown at point A on a corner and V_2 at point B. [3]

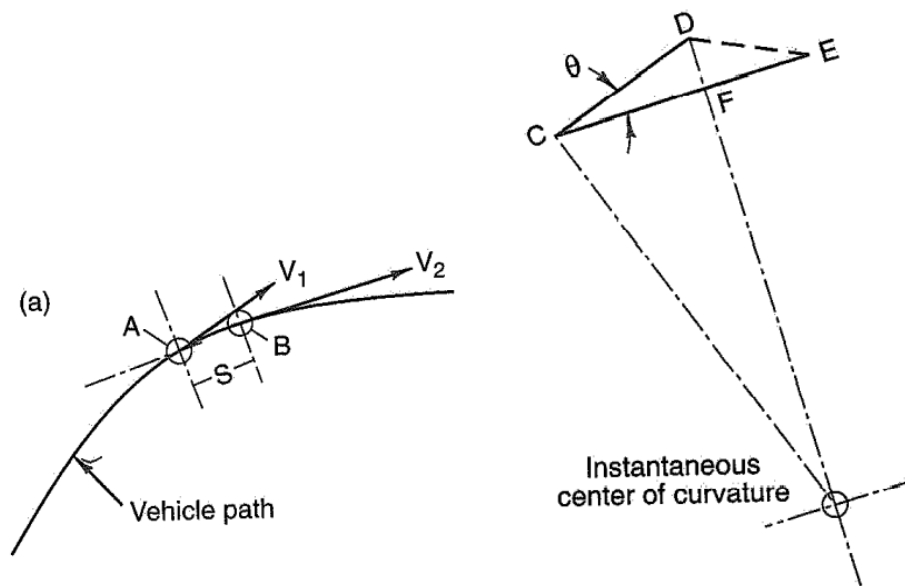


Figure 1.6 (a, left) and (b, right). Vehicle changing in both speed and direction. [3]

Figure 1.6(b) is a distance diagram. If the vehicle, now starting at point C, had continued at the same velocity represented by vector V_1 (that is, if it had been no acceleration or velocity change) it would have reached point D in the time, t , whereas in actual fact it reached point E (CD being parallel to V_1 and CE being parallel to V_2). Thus, during the time, t , the vehicle must have experienced a speed change in an inward direction (toward the center of curvature) of DF/t . This rotates the vector V_1 through an angle θ . During the same time a speed change must have occurred in the FE direction, of FE/t . These are called the normal (or centripetal) and tangential components of the acceleration; to the driver they are the lateral and the longitudinal accelerations experienced in rotating and lengthening the velocity vector from V_1 to V_2 in t . The total acceleration vector is DE/t , which lies along the dotted line in the figure. The main important point here is that to maintain a curved path on the ground the vehicle must be moving sideways as well as forward, and lateral acceleration is the change in lateral speed with time to achieve this. [3]

Now, to consider the process in which a vehicle follows a curved path of constant radius, figure 1.7 could be helpful. Here a vehicle enters a turn at the point t_0 (also taken as time zero). At this point the vehicle has a velocity of V . Assume that the

turn is taken at constant speed. A short time, say, 1 second, after the vehicle has entered the turn, it will have reached point t_1 , and, in 2 seconds, t_2 . The question is: What does it take in the way of lateral velocity for the vehicle to stay in the curved path? At t_1 , the vehicle must have moved laterally a distance, d_1 . To do this required a lateral velocity of $d_1/1\text{sec}$. At t_2 the vehicle must have moved laterally a distance d_2 , which from observation is about three times the lateral distance required at t_1 . The lateral velocity to move d_2 is $d_2/2\text{sec}$, which is greater than the lateral velocity required to stay in the path at t_1 by a factor of about $(d_2/2\text{sec}) / (d_1/1\text{sec}) = \left(\frac{d_2}{d_1}\right) * \frac{1}{2} = (3/1) * \frac{1}{2} = 1.5$. [3]

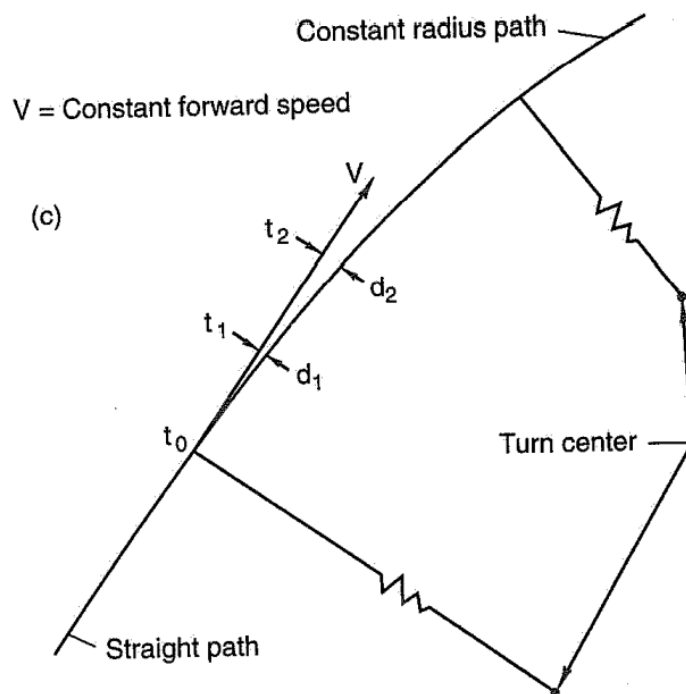


Figure 1.7. Lateral acceleration. [3]

Thus the lateral velocity is increasing with time. An increase in velocity with time is an acceleration, here, a lateral acceleration.

The lateral acceleration is given by the relation V^2/R where V is the speed along the path and R is the instantaneous radius of the path. In this field, it is common to express the acceleration in terms of the acceleration experienced by a falling body in the earth's gravitational field. Lateral acceleration then becomes V^2/gR and longitudinal acceleration, $\Delta V/g\Delta t$, both in "g" units. [3]

The rotation and length changes of the acceleration vector, as the vehicle progresses along a circuit, has led to the concept of the “g-g” diagram, which will be discussed in later chapters. Basically is the recording of the outputs of longitudinal and lateral accelerometers of the vehicle, and after that, a plot can be made of driver/vehicle performance. An example of a “g-g” diagram is shown in figure 1.8.

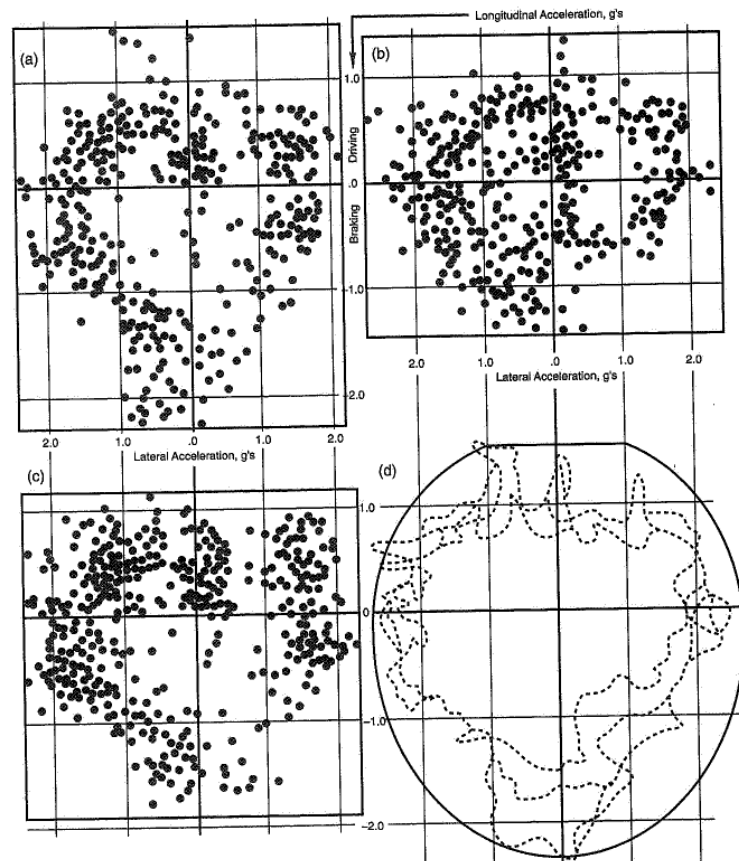


Figure 1.8. G-G measurements on a Grand Prix car. [3]

2. Tire behaviour

The forces for accelerating the car in the horizontal plane originate principally at the tyres. An understanding of the tyre behavior is one of the keys to achieve the largest “g-g” diagram.

Tyres support the vehicle weight, and any other vertical forces such as aerodynamic or the result of road banking. The interaction between the tyres and the road supply the tractive, braking and cornering forces for maneuvering (as plotted on the “g-g” diagram). The tyres also supply the forces used for controlling and stabilizing the vehicle and for resisting external disturbances from road and wind. For these beneficial effects the tyre demands a price in rolling resistance and induced forces in the drag direction. The tyre generate steering torques which give rise to centering effects in the steering system and related torques felt at the steering wheel by the driver. [3]

2.1. Lateral Force

In SAE J670, vehicle dynamics terminology, a lateral force arises from the “center” of the tire's contact with the road, is in the horizontal plane of the road, and is perpendicular to the direction in which the wheel is steered, with no pitch or camber. Some experiments have been conducted to understand tire behavior. [3]

2.1.1. The model tyre

The model tyre described in this chapter was used for the first time by Chevrolet R&D as an instructional tool in the 1960. This tyre is made by sandwiching a sheet of hard rubber between two metal disks as shown in Figure 2.1 [3]

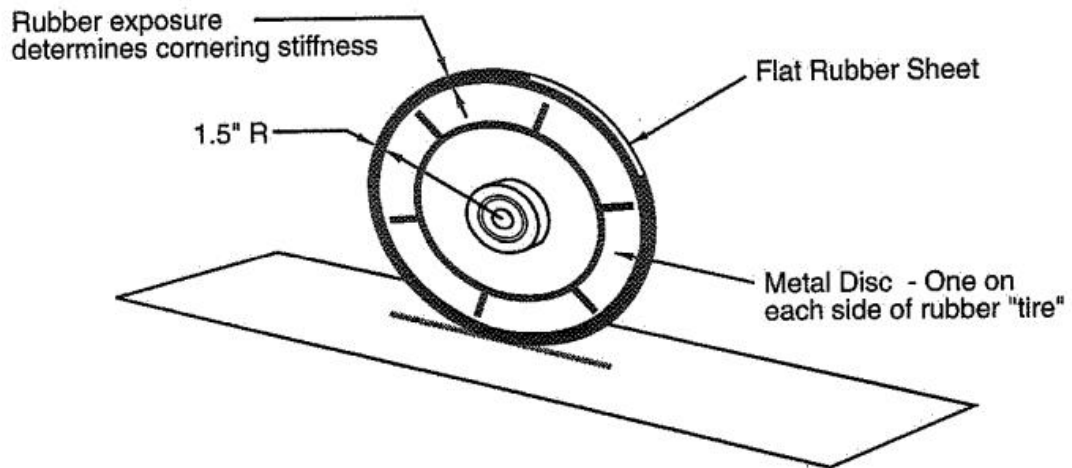


Figure 2.1. Model tyre. [3]

2.1.2. Static lateral stiffness and sliding

If the tyre model is pressed lightly on a table with a constant vertical force, it is easy to see that with lateral pressure due to a small lateral force, the rubber deflects and the wheel moves a little laterally.

Lateral Stiffness is a lateral spring rate. The force required to slide a tyre is a function of the coefficient of friction μ , which is the lateral force divided by the vertical force. This is approximately constant for the model tyre over a range of loads.[3]

2.1.3. Slip angle

Every vehicle, without apparent cause of slippage, tends to move laterally in one direction, and forms an angle, δ , with the perpendicular to its axis of rotation, i.e. with the path of travel, this angle is called slip angle. This is due to the deformation of the tyre tread and sidewalls when the tyre is subjected to transverse or lateral stresses. A lateral force F_1 acts on the tyre in a direction perpendicular to the plane of the wheel, due to centrifugal acceleration during cornering or wind.

The slip angle, δ , is formed by the two steering components, the theoretical movement 1 and the real or effective movement 2. [6]

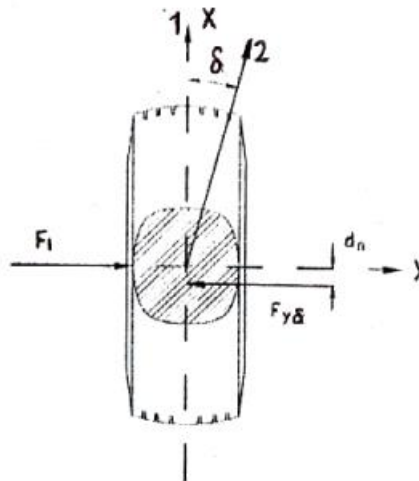


Figure 2.2. Slip angle. [6]

What happens in a straight? When the vehicle is subjected to a lateral force F_1 , the part of the tyre in contact with the ground is held in position by the adhesion of the ground, while the rest of the wheel tends to move due to the force acting on it, which causes a transverse deformation in the tread produced by the resultant of both components and causes the wheel to follow a different trajectory to the desired one. [6]

At low speeds or when the lateral force component is small, the deformation of the wheel is absorbed by the transverse stiffness of the tyre which cancels out the effect of the slip angle; but at high speeds or when the lateral force component is strong, the slip angle becomes noticeable and constant steering correction is necessary to keep the vehicle straight. [6]

And during cornering? when cornering, the front wheels take the curvilinear trajectory imposed by the steering, but due to the action of the disturbing forces acting on the car, such as centrifugal force, wind, traction, braking, a lateral displacement is produced, which tends to deviate it from the theoretical trajectory to be followed. In this case the lateral displacement of the wheels modifies the centre of rotation of the vehicle where the centre of gravity describes a divergent trajectory with respect to the theoretical direction, which tends to go out of the curve. [6]

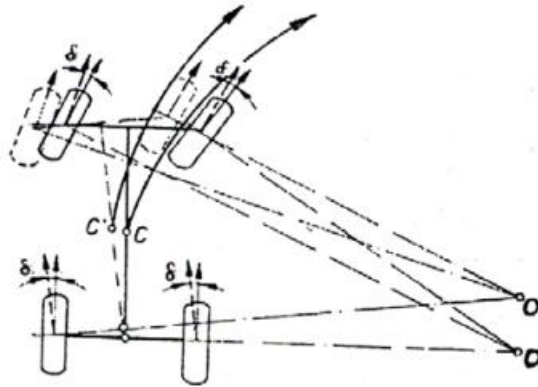


Figure 2.3. Effect of slip angle when cornering. [6]

2.1.4. Tyre deformation under vertical load

The consideration of the normal load acquires greater importance in the study of stability and trajectory control of the vehicle, if the effect of load transfer from one wheel to another is taken into account, both during cornering and under the effects of other lateral actions. Figure 2.4 shows the deformation of the tire under the action of the normal load G_r applied to the wheel. The deformation of the tire is expressed by the decrease of the distance from the wheel axle to its bearing surface, compared to the free radius r_0 of the unloaded tire by a value equal to h . [6]

In this case, between the tire and the supporting surface is created the contact surface on which the normal reactions of the road act, balancing the load transmitted by the wheel. The magnitude h of height variation in the tire profile under load is called the normal tire deformation. [6]

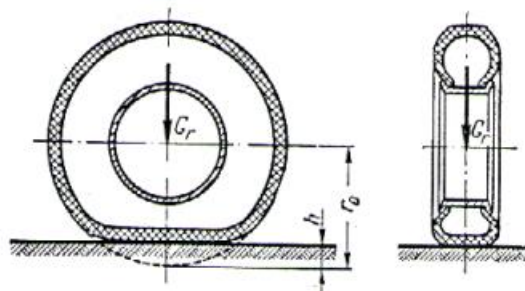


Figure 2.4. Tyre deformation under vertical load. [6]

During the deformation of the tyre, the volume occupied in the tyre by the air decreases somewhat, however, compared to the total volume of air, this change is negligible, and it is therefore assumed that the deformation has no influence on the air pressure in the tyre. The ratio:

$$\lambda_n = \frac{G_r}{h}$$

Where λ_n is the average (reduced) stiffness coefficient of the tyre in normal direction, it is one of the most important tyre parameters. [6]

Its value depends mainly on the air pressure in the tyre (the lower the pressure, the higher the normal tyre deformation and the lower the stiffness coefficient), and on the tyre dimensions, structure and materials used for tyre construction (the stiffer the outer layer of the tyre, the higher the value of the stiffness coefficient). [6]

There is no direct proportionality between the normal deflection h and the load G_r acting on the wheel. As the load increases, the deformation increases more and more slowly and, therefore, it is possible to speak about a certain value of λ_n only in the case of a relatively small range of variation of the load G_r . [6]

2.1.5. Lateral deformation of the tyre

If a wheel in a vertical position, as shown in figure 2.5, loaded with a radial force and moving in the plane of its rotation in the direction indicated by the arrow V , is given the lateral force F_1 acting along the axis of the wheel, this will cause the tyre profile to deform and the shape of its contact with the road to change. [6]

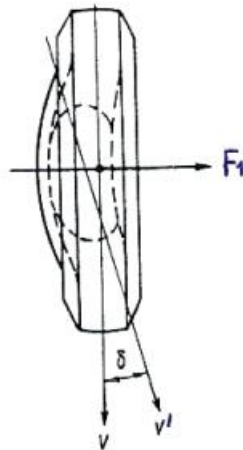


Figure 2.5. Lateral force applied on a tyre. [6]

The tyre profile is not only deformed in the contact area, but also outside its limits. This causes the so-called lateral deflection of the tyre, which is expressed by the fact that the wheel deviates from the initial direction and starts to move under a certain angle δ to it, as shown in figure 2.5 with the arrow V' . The angle δ is called the angle of lateral deflection. The ratio: [6]

$$K_d = \frac{F_1}{\delta}$$

Where K_d is the coefficient of resistance to lateral deflection.

Tyre structure, tyre size and tyre pressure have a major influence on the lateral deflection resistance of tyres; increasing air pressure increases the lateral deflection resistance. The lateral deflection resistance equation, which determines the linear dependence between the deflection angles and the lateral forces acting on the wheel, is only valid up to the point where deflection occurs without lateral slippage of the tyres. [6]

This corresponds to very limited deflection angles not exceeding 3-5° for passenger car tyres. The larger the angle δ , the greater the sideways slip of the tyre begins, which quickly increases until it becomes skidding, when the lateral force of adhesion between the tyre and the road is fully utilised. [6]

2.1.6. Contact patch with slip angle

The force F_1 applied to the wheel axle and $F_{y\delta}$ applied in the contact path produce the transverse deformation of the tyre (figure 2.6)

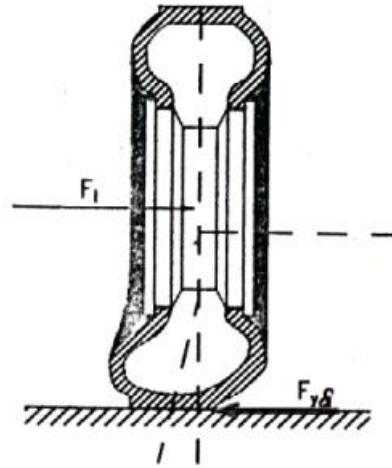


Figure 2.6. Lateral deflection. [6]

When rolling the tyre under these conditions, a point on its periphery will move slightly from its undeformed position as it approaches the contact area Z1 (figure 2.7). This displacement is due to the deformation of the carcass only. [6]

When, following the rotational movement of the wheel, it enters the contact area (position M1), it will continue to move transversely due to the deformation of the carcass, tread and a small slip, until it reaches a position M2, at the edge of the area Z2. In this zone the tyre-to-ground pressure is sufficient to prevent any slippage. [6]

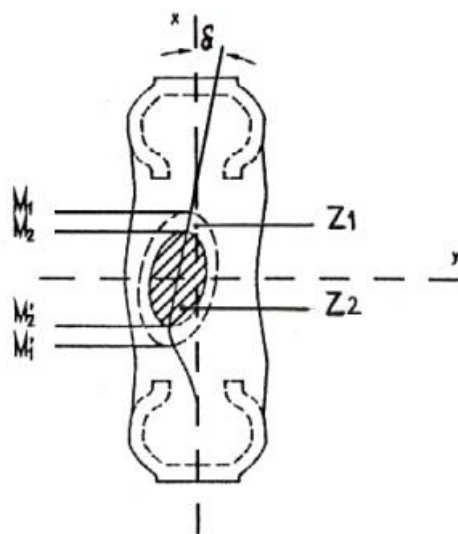


Figure 2.7.[6]

During its movement through the zone Z3, the adhesion imposes a straight trajectory $M2 - M2'$ which forms an angle δ with the plane of the wheel. On leaving the Z2 zone, it again slides in the opposite direction into the Z1 zone and then continues to move out of the contact patch until it returns to its undeformed position in the tyre. For small values of slip angle there is practically negligible slip and the straight segment $M2 - M2'$ extends over the entire contact area. As the slip angle increases, the slippage increases, being much more accentuated in the rear part of the contact patch. [6]

2.1.7. Measurement of tyre lateral force data

Tyre force and moment data are measured by constrained test, but there are still many problems concerning the accurate measurement of tyre data, for example:

- Tyre wear and use of multiple tyre samples (to obtain data on one tyre design)
- Lack of control of tyre temperature.
- Accurate control of test conditions.
- Test machine limitations in ranges of load, power, slip/camber angles, etc.
- Treatment of measured test data and data reduction.

One of the most important problems in vehicle dynamics is obtaining enough reliable data on tyre designs of interest. [3]

Figure 2.8 shows a plot of lateral force versus slip angle for a typical racing tire. This data was recorded at the Calpsan Tire Research Center on a simulated flat, dry road. The tire rolls freely and the curve represents a single load. The peak of the curve can remain at a constant value or slowly decrease as indicated. In dry conditions, racing tires typically achieve their maximum lateral force at slip angles close to $3-7^\circ$. On a wet surface, the peak is generally slower and the lateral force decay after the peak is faster. [3]

P215/60 R15 Goodyear Eagle GT-S (shaved for racing) 31 psi.
 For a given load, in this case 1800 lb.

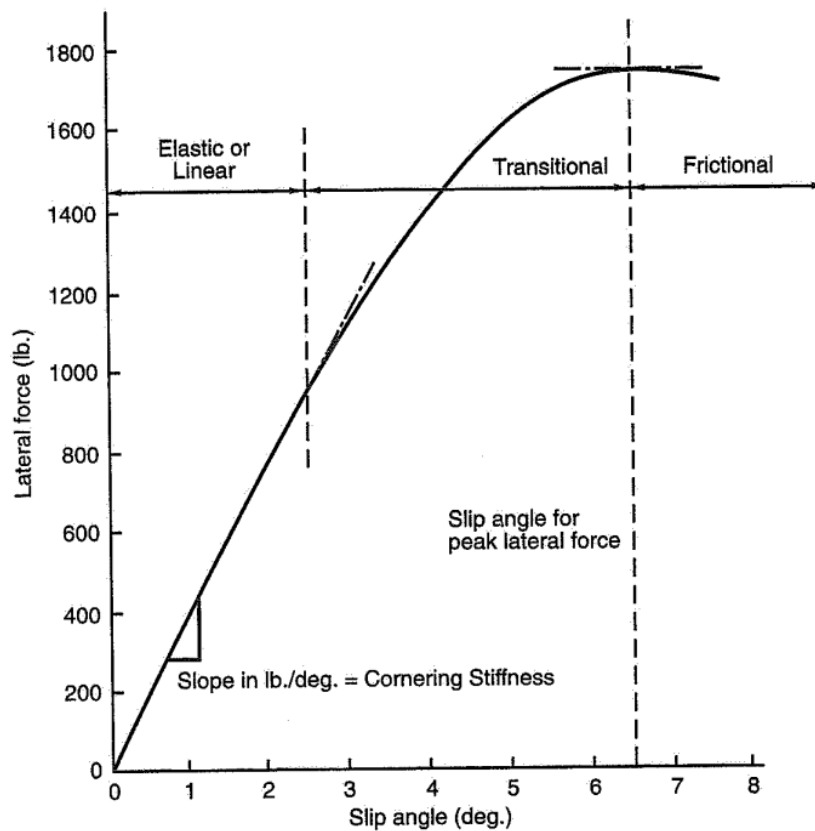


Figure 2.8. Lateral force v slip angle for a racing tyre.[3]

When a particular tyre is tested at different loads, the lateral force curves appear as in Figure 2.9. As the load increases, the peak lateral force occurs at a somewhat higher slip angle and the cornering stiffness (slope in the elastic range) increases as well.

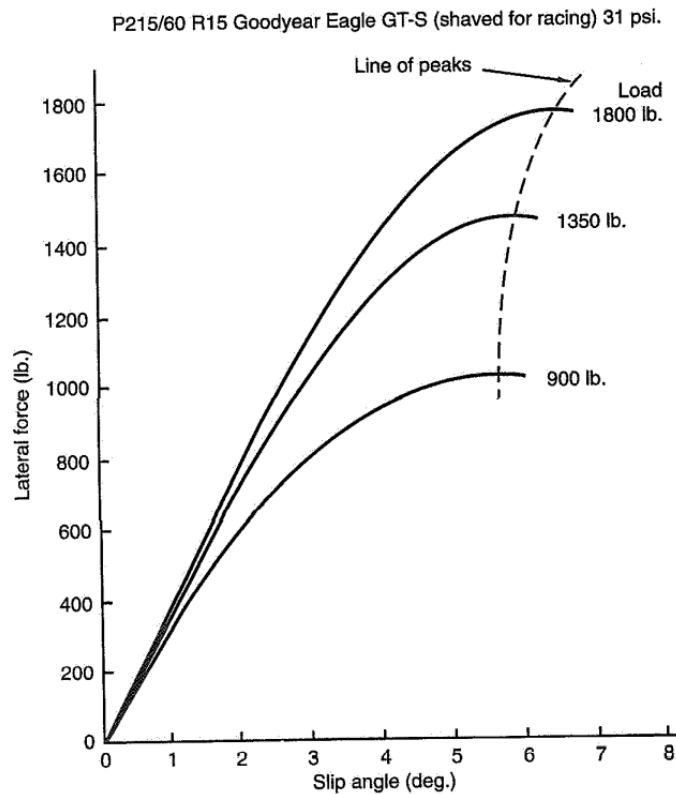


Figure 2.9. Lateral force v slip angle for different loads.[3]

2.1.8. Tire load sensitivity

At and beyond the peak, most of the print is sliding and the lateral force is the result of the friction between the tyre and the road surface. The friction coefficient is defined as

$$\mu = \frac{\text{Frictional force between two bodies}}{\text{Normal force between two bodies}}$$

The amount of lateral force in relation to the load is:

$$\frac{\text{Lateral force}}{\text{Load on tire}} = \text{Lateral Force Coefficient}, \frac{F_y}{F_z} \quad (F_z = G_r)$$

And curves such as those of figure 2.9 can be replotted as in figure 2.10.

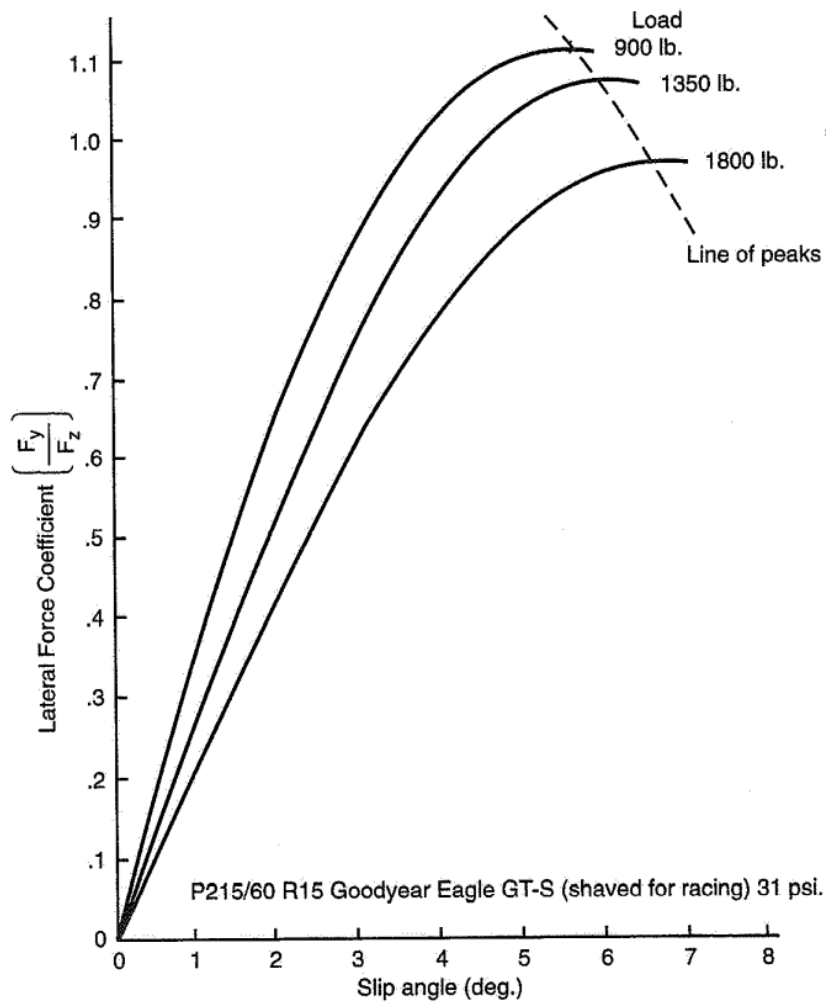


Figure 2.10. Lateral force coefficient v slip angle. [3]

It is easy to see that the peaks of all of the load curves are more closely together when plotted in this fashion. If they all reached exactly the same value, the peak lateral force for any load would be (lateral force coefficient) x (load on tyre).

In reality, the coefficient of maximum lateral force (or the coefficient of lateral friction) is usually higher for the lightest load or, conversely, decreases as the load increases. This effect is known as "tire load sensitivity". The lateral friction coefficient is independent of the speed, and can be increased with a more "sticky" rubber compound and by ensuring proper operating temperatures. For a current racing tire, the coefficient can reach 1.8 under high loads. [3]

The tire performance transition zone varies depending on tire design parameters. It can be gradual or more abrupt. Some tires have soft characteristics that warn a lot, while other tires suddenly become "loose". Tires that achieve higher

coefficients may let go faster as more of the print has been utilized for elastic distortions [3].

Tire load sensitivity has a major impact on the balance of a racing car at the limit. As the vertical load on tires changes due to cornering, acceleration or braking, tire performance is constantly changing. [3]

It is important to note also the variation in lateral force with load for a given slip angle as shown in figure 2.11 for modern radial tyres.

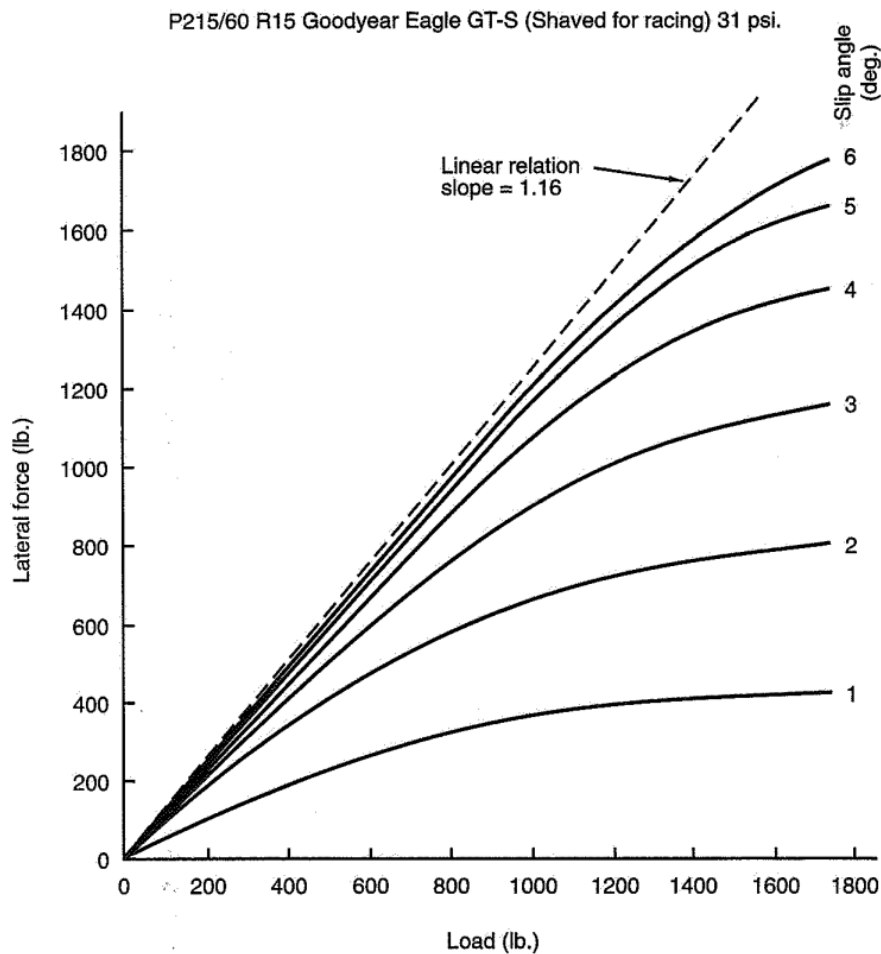


Figure 2.11. Lateral force v load for a given slip angle.[3]

2.2. Aligning torque and pneumatic trail

Aligning torque describes the tendency of a tire to rotate about a vertical axis through the "center" of the print (the origin of the tire's axis system). At low and medium slip angles, the tire tends to align its course with its trajectory. The alignment torque results from the shape of the print. Figure 2.12 shows that the

shape of the distortion in the print is roughly triangular and not symmetrical to the fore-aft center of the print. The elastic distortion in the print increases from front to back and this leads to an uneven distribution of lateral force across the print. This uneven distribution results in an aligning torque. [3]

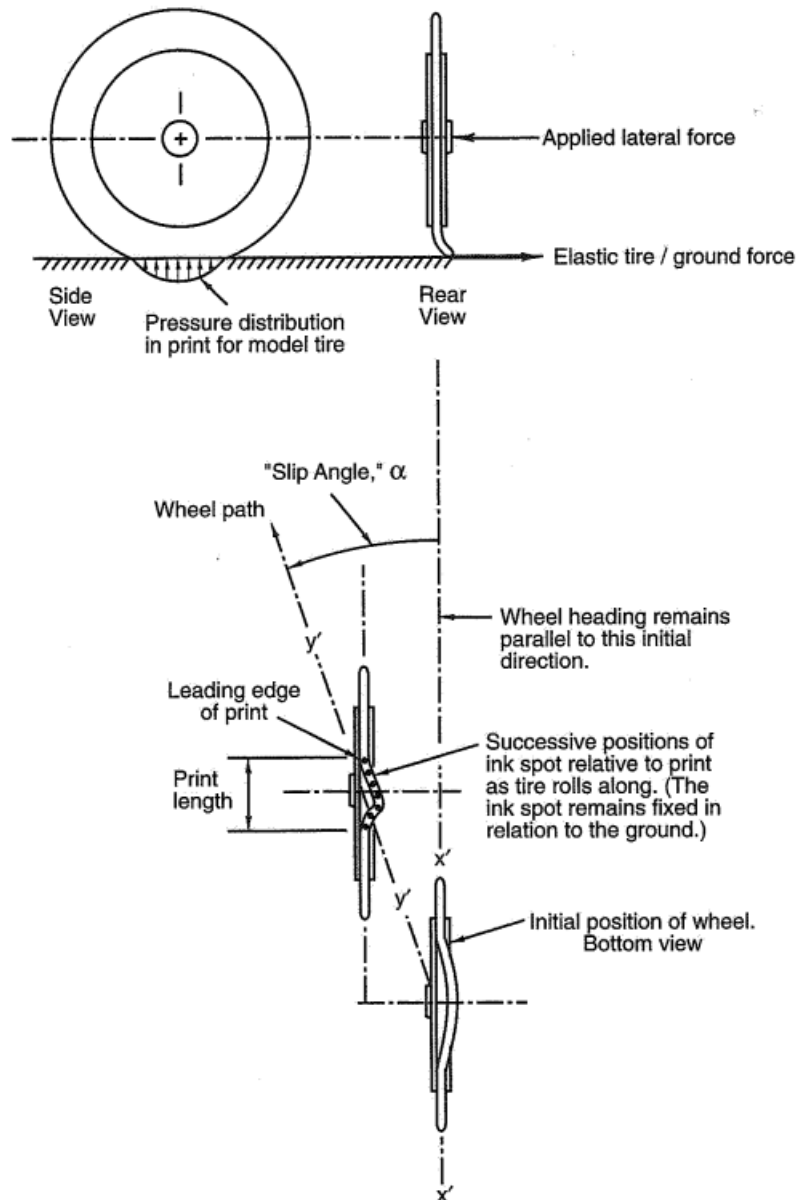


Figure 2.12. Mechanism of tire lateral force in elastic range.[3]

Pneumatic trail is the distance from the fore-aft center of the print to the center of action of the lateral force. The tire aligning torque is the lateral force times the pneumatic trail. [3]

2.2.1. Linear range – low slip angles

From figure 2.13, the higher stresses in the aft part of the print work to reduce the slip angle. This is true in the linear range of tire performance.

2.2.2. Non linear range – high slip angles

At high slip angles, the rear of the print is sliding laterally along the ground. This reduces the amount of stabilizing aligning torque. At the friction limit the aligning torque is near to zero and could be even negative. This means that the tire is sliding and it no longer has any tendency to line up with its path. [3]

2.2.3. Mechanical trail, pneumatic trail and steering torques

Trail is represented in its simplest terms by the wheels of a shopping trolley. The tire print trails behind its steering pivot. Automotive steering systems are designed in the same way, the caster angle provides a mechanical trail as shown in Figure 2.13. One more way to get a mechanical trail is to move the kingpin forward of the center of the wheel. This kingpin offset is shown in Figure 2.14. [3]

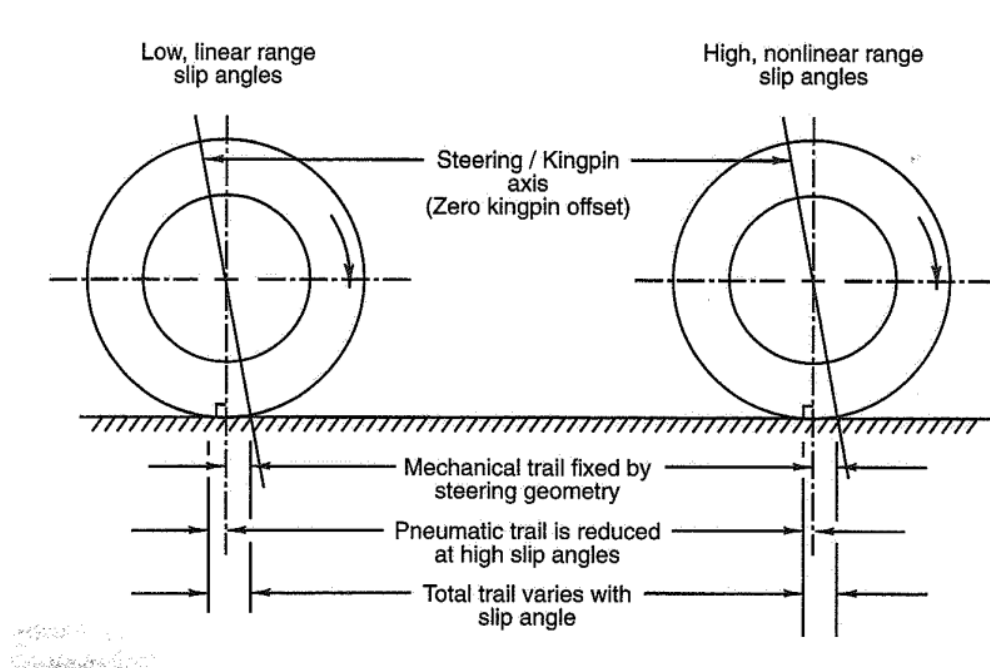


Figure 2.13. Mechanical and pneumatic trail.[3]

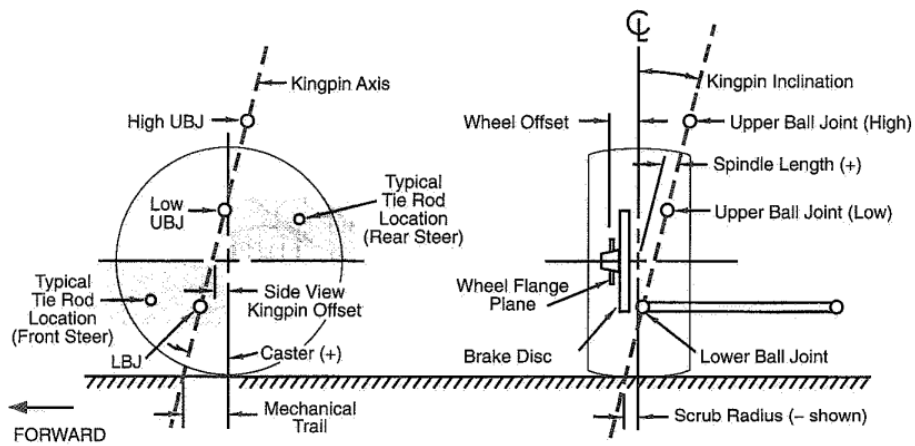


Figure 2.14. Kingpin geometry.[3]

If all of the tire's lateral force at the print were concentrated directly under the axle, the steering torque would be the mechanical trail multiplied by the lateral force. The aligning torque can also be considered as a propulsion effect. Pneumatic trail is always available; varies with tire operating conditions. The sum of the mechanical trail plus the pneumatic trail multiplied by the lateral force results in the steering torque at the kingpin. When the mechanical trail is small, the tire aligning torque dominates the steering torque. Some modern cars have so much mechanical trail that the effect of aligning torque is minimized. This provides a linear relationship between lateral force and kingpin torque without aligning torque information reaching the driver. Maximum warning of breakaway would occur if all of the steering torque came from the pneumatic torque. Therefore, the great fall-off in aligning torque shown in Figure 2.15, which occurs before the lateral force peak, would tell the driver that the limit has been reached.

A balance between the two gives the skilled driver some warning that the front wheels are nearing the limit, while maintaining reasonable steering torques. [3]

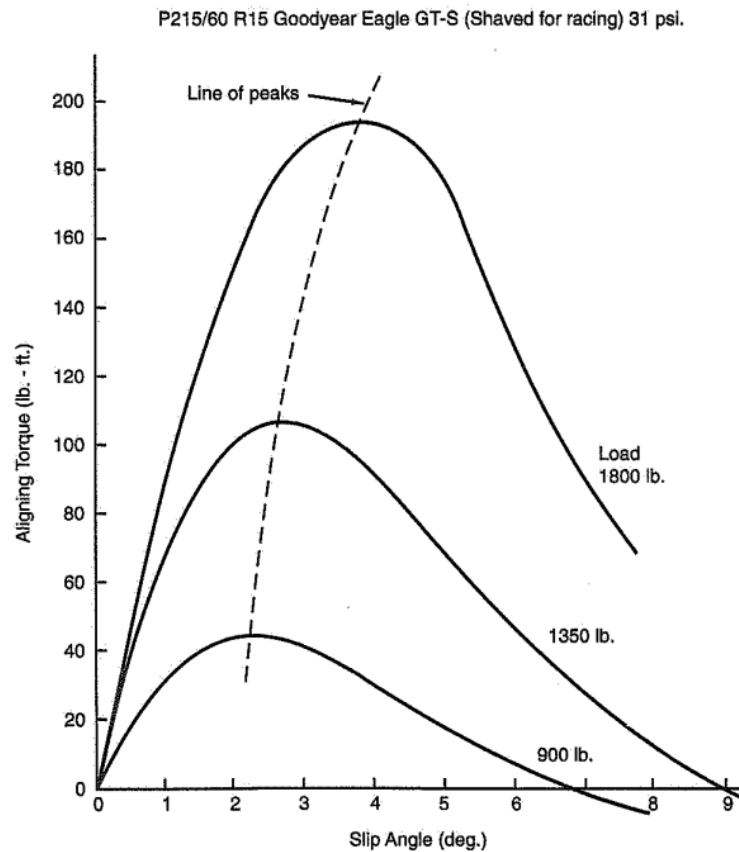


Figure 2.15. Aligning torque v slip angle for different loads.[3]

2.2.4. Pneumatic trail and skid warning

The following numerical example show the significance of pneumatic trail in warning the driver of impending breakaway. It is based on the tire data shown in figures 2.9 and 2.15. This tire size would be used on a large front-engined sports car of some 3500 lb. Gross weight with 1800 lb on the front wheels. Consider that this vehicle is set up with a front anti-roll bar to lift the inside front wheel approach to max lateral. Since the inside wheel is “dancing” along the road producing negligible lateral force and aligning torque, the calculation can be performed for the outside front wheel only, now carrying the full 1800 lb. Maximum lateral force for this tire occurs at $\alpha = 6.5^\circ$ (figure 2.9) at a maximum lateral acceleration of the vehicle of about 0.97g. [3]

We are interested in estimating the steering wheel rim force due to pneumatic and mechanical trail over a small range of slip angle and lateral acceleration prior to breakaway. We have chosen to use slip angles of 5.5° to 6.5° corresponding to

0.94 to 0.97g. The calculation is performed for mechanical trail offsets of the Kingpin of 0.00", 1.25", and 2.49" which correspond respectively to 0°, 6°, and 12° of caster angle on a front tire radius of 12". For this example, offset is used to avoid other components of torque which arise with caster angle and which are not relevant for the present example. Finally the calculation assumes no steering boost, a steering system with low friction and a steering wheel radius of 7". [3]

The results given in figure 2.16 show that a reduction of steering wheel rim force of nearly 30% occurs during the last 1° of slip angle before breakaway when all of the rim force comes from pneumatic trail. When mechanical trail is added, the percent reduction is substantially diluted. The two dashed curves show that rim force due to mechanical trail alone actually increases slightly due to the increase in tire lateral force as maximum lateral is approached. [3]

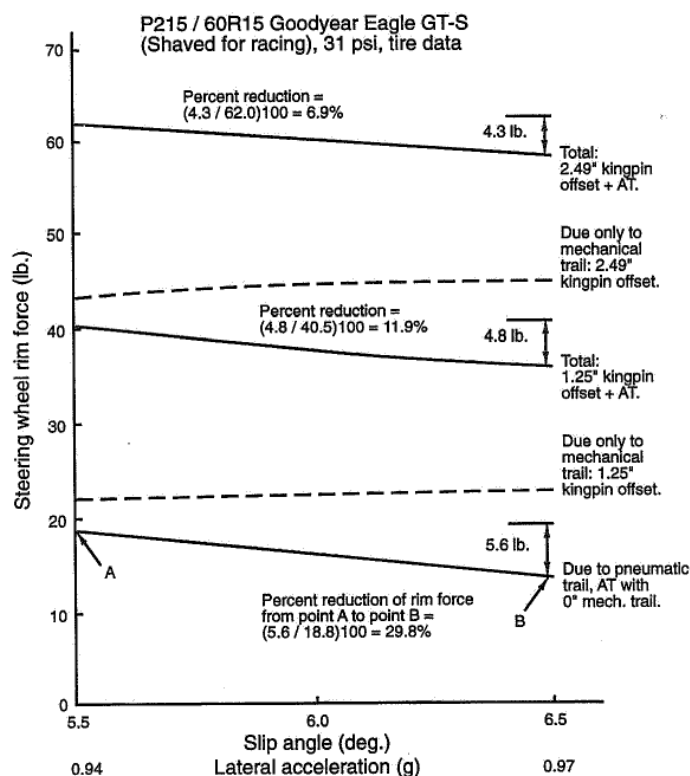


Figure 2.16. Rim force due to pneumatic and mechanical trail.[3]

2.3. Tractive force

In real rolling, in traction, when reaching the contact area, the points of the perimeter remain on the ground for a while, and then slide and leave the contact area, so it cannot be said that the point of the perimeter (A) has a sliding speed, because when it reaches the ground it does not slide completely, but there is a moment when it is adhered and another at the end when it slides. [7]

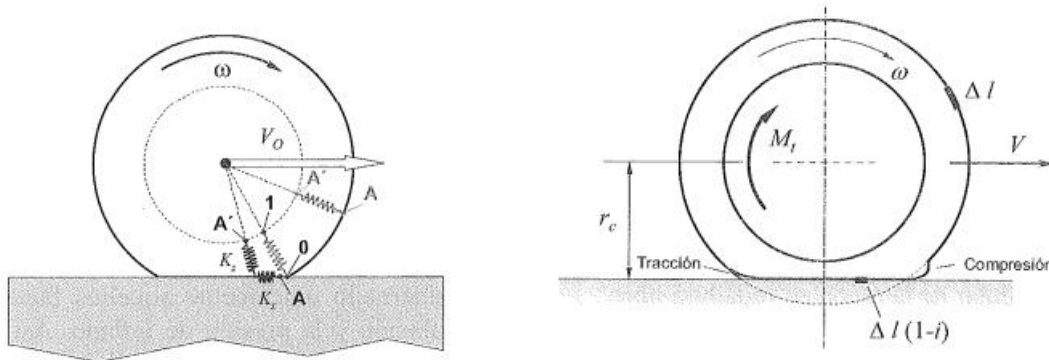


Figure 2.17. Tire under a tractive force. [7]

When the point touches the ground (Figure 2.17), the longitudinal spring is stretched, which means that the wheel rotates faster than the circumference of the tread, so that $\omega * r$ is greater than the linear velocity of the wheel V_0 . Therefore the velocity of point A is: $V_A = \omega * r - V_0$. [7]

The velocity of point A, which, from a reference system linked to the wheel centre, will have a backward direction, is known as the pseudo-sliding velocity, since point A remains stationary on the ground for a while and then, when it is at the end of the contact area, it slides. With this, it is defined as pseudo-sliding: [7]

$$\text{Pseudo-sliding} = \frac{\omega * r - V_0}{\omega * r}$$

Thus, for example, when a traction torque is applied with the vehicle stationary and the wheel begins to acquire an angular velocity ω and starts to slip without moving forward, $V_0 = 0$, the pseudo-slip is equal to 1. With the above explanation we can see, that due to these deformations, the tyre behaves as if its radius were smaller than that corresponding to free rolling, so that the speed at which the wheel axle moves, V , is less than the theoretical one, $\omega * r$, where r is the radius of the tyre in the free rolling condition. This speed differential, previously defined

as pseudo-slip, is also known as slip(*i*). For SAE, the definition of slip is as follows[7]

$$i = \frac{\omega - \omega_0}{\omega_0}$$

Where ω_0 is the angular velocity in free rolling and ω is the real angular velocity.

However, the most used formula for the slip is the following:

$$i = \frac{V_A - V_0}{V_A}$$

Where V_0 is the wheel axle translational velocity and V_A is the ground contact point speed.

With this expression, the slip takes values between 0 and 1. It is zero when the wheel is in free-rolling ($V_0 = V_A$) and one when the wheel slips ($V_0 = 0$ and $V_A \neq 0$).

[7]

If the tyre is viewed as a rigid body of variable radius, the above expression can be extended as follows, considering that the tyre has an effective radius of spin.

$$i_t = 1 - \frac{V_0}{\omega * r_c} = 1 - \frac{r_e}{r_c} \quad r_e < r_c$$

Where:

V_0 = wheel axle translational speed.

ω = angular speed of the wheel.

r_e = free-rolling effective radius of the tire ($r_e < r_c$)

r_c = radius under load of the tire.

i_t = longitudinal slip in traction

The pseudo-longitudinal tyre slip is related to the longitudinal forces generated when the tyre comes into contact with the ground and, consequently, to the ratio between the force F_x and the vertical load F_z gravitating on the tyre, the so-called μ_{xt} traction grip coefficient.[7]

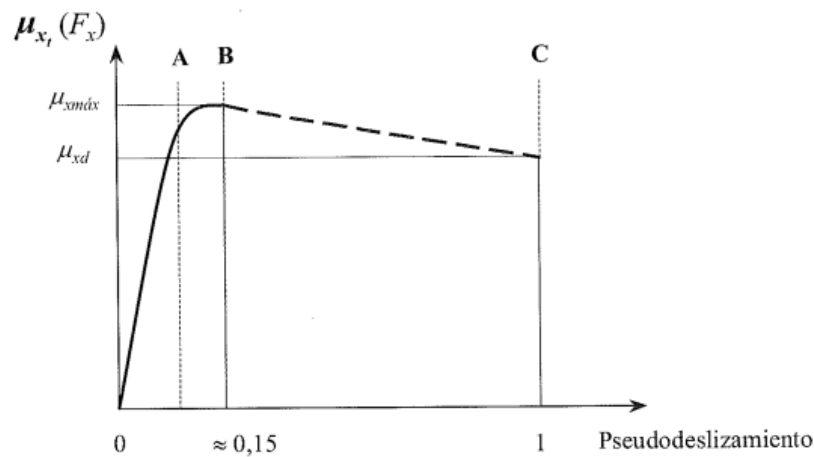


Figure 2.18. Changes on the longitudinal stress coefficient with slip.[7]

The coefficient of adhesion of the tyre is related to the pseudo-slip as shown in figure 2.18, a curve obtained experimentally. Although this relationship depends on the type of surface on which the tyre rolls, three zones and two characteristic points can be distinguished in all the graphs. In the first zone, until the maximum coefficient of adhesion is reached (up to point A), the relationship between the two is linear. This is because, when the pseudo-slip is small, it appears due to the elastic deformation of the tread. When the pseudo-slip increases due to the effect of the increase in torque, the pseudo-slip is no longer linear, due to the skidding on the tread surface which starts at the rear of the tread, the area of lowest tread pressure. In this situation a slight increase of the torque means that the slip is spread over the whole track, so that the pseudo-slip increases to unity ($V = 0$, $\omega \neq 0$), i.e. pure slip occurs, and the coefficient of tractive effort drops to the dynamic value μ_{xd} . The transition from the point of the value μ_{max} to the point of μ_{xd} happens very quickly (dashed line in figure 2.18).

As can be seen in Figure 2.18, the value of the maximum coefficient of adhesion of a tyre rolling on a hard surface is obtained when the pseudo-slip value is between 0.1 and 0.3, with the lowest values corresponding to high performance tyres. [7]

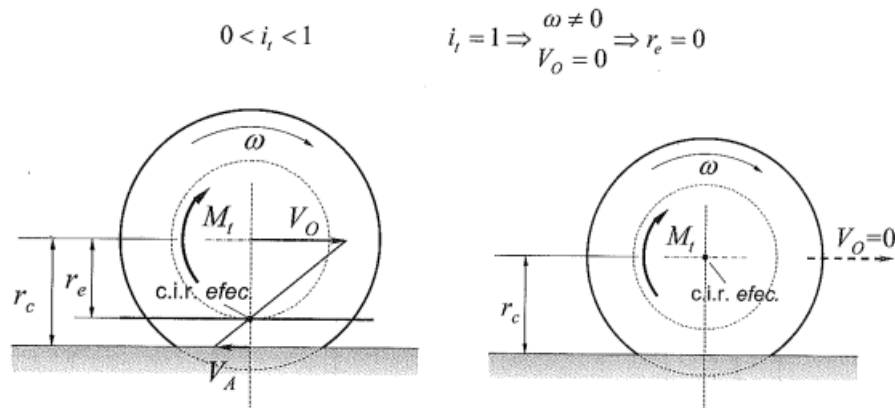


Figure 2.19. Relation between the effective radius and the movement of the wheel. Slippage (picture at the right).[7]

2.4. Braking force

During the braking process, a phenomenon similar to that described for traction occurs. In this case, when a torque is applied in the opposite direction to that of wheel rotation, the tread stretches when it contacts the ground at the front. This means that the linear speed of the wheel is greater than the product of $\omega * r$. Pseudo-slip under braking is the ratio of the pseudo-slip speed divided by the linear speed of the wheel V_0 . [7]

$$\text{Pseudo-sliding} = \frac{V_0 - \omega * r}{V_0}$$

So when, for example, during braking the wheel locks and $\omega = 0$, as there is a travel speed, the pseudo slip becomes equal to 1.

How braking affects the periphery of the tyre can be seen in figure 2.20, where the deformation of the tread produces stretching in the area that comes into contact with the ground while the exit part is compressed.[7]

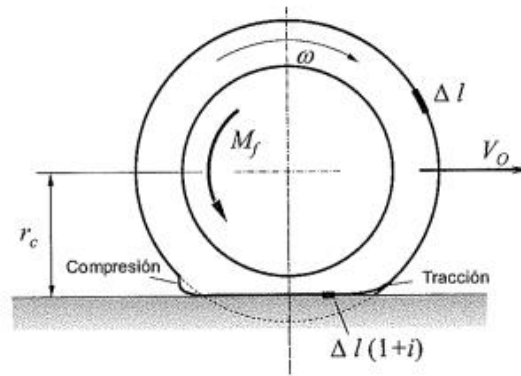


Figure 2.20. Tire under braking stress. [7]

Therefore, due to the deformations, the tyre rolling occurs as if the tyre diameter is larger than the free rolling diameter under the same load. The displacement velocity of the wheel axle (V_0) will be higher than that corresponding to free rolling $\omega * r$. As for traction, this effect is defined as slippage, and its definition is as follows: [7]

$$i_f = \frac{V_0 - V_A}{V_0}$$

Again, considering the wheel as a rigid solid of variable radius, it is obtained:

$$i_f = 1 - \frac{\omega * r_c}{V_0} = 1 - \frac{r_c}{r_e} \quad r_e > r_c$$

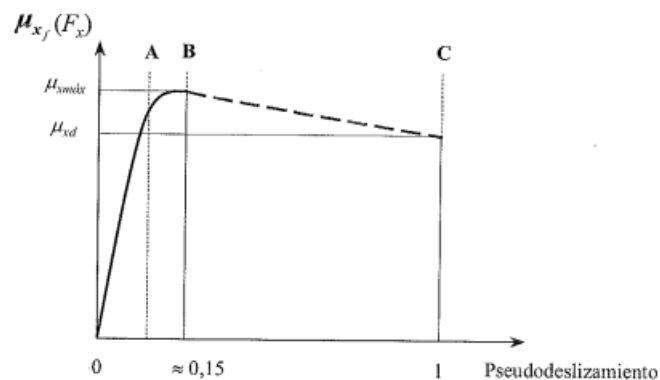


Figure 2.21. Changes on the longitudinal stress coefficient with slip while braking.[7]

The relationship between the coefficient of longitudinal adhesion under braking and the pseudo-slip is similar to that described for traction, as can be seen in Figure 2.21. [7]

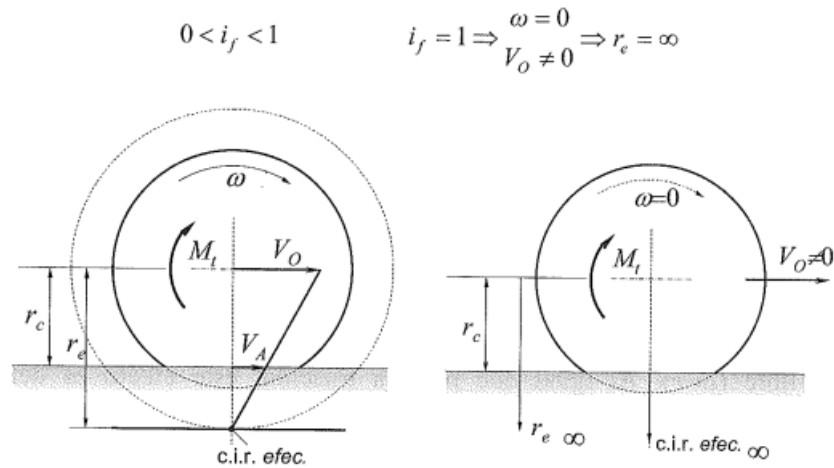


Figure 2.22. Relation between the effective radius and the movement of the wheel. Wheel locked (picture at the right).[7]

2.5. Camber effects

Camber angle, Φ , is defined as the angle between a tilted wheel plane and the vertical. The camber is positive if the wheels leans outward at the top relative to the vehicle, or negative if it leans inward. [3]

2.5.1. Camber force

A cambered rolling pneumatic tyre produces a lateral force in the direction of the tilt. When this force occurs at zero slip angle, it is referred to as camber thrust. A lateral force component due to camber can also occur at slip angles other than zero. When a stationary tyre is pressed down onto the road at a camber angle, the center plane of the print is curved (see Figure 2.23(a)). When the tire is rolled at zero slip angle (b), a point entering the print is constrained by the road to move through the print on a straight path defined by the direction of motion. Thus the road applies forces to the tire which tend to remove the curvature in the stationary (nonrolling) tire print. The sum of these forces is the camber thrust. [3]

It is possible to compare the lateral force produced by camber angle to that produced by slip angle. Because of the shape and size of the print distortion patterns, the lateral force generated in the linear (small angle) range by one degree of slip angle is greater than that generated by one degree of camber. For

traditional bias tires the cornering stiffness ($\Delta F_y/\Delta\alpha$), is generally five to six times greater than the camber stiffness ($\Delta F_y/\Delta\Phi$). [3]

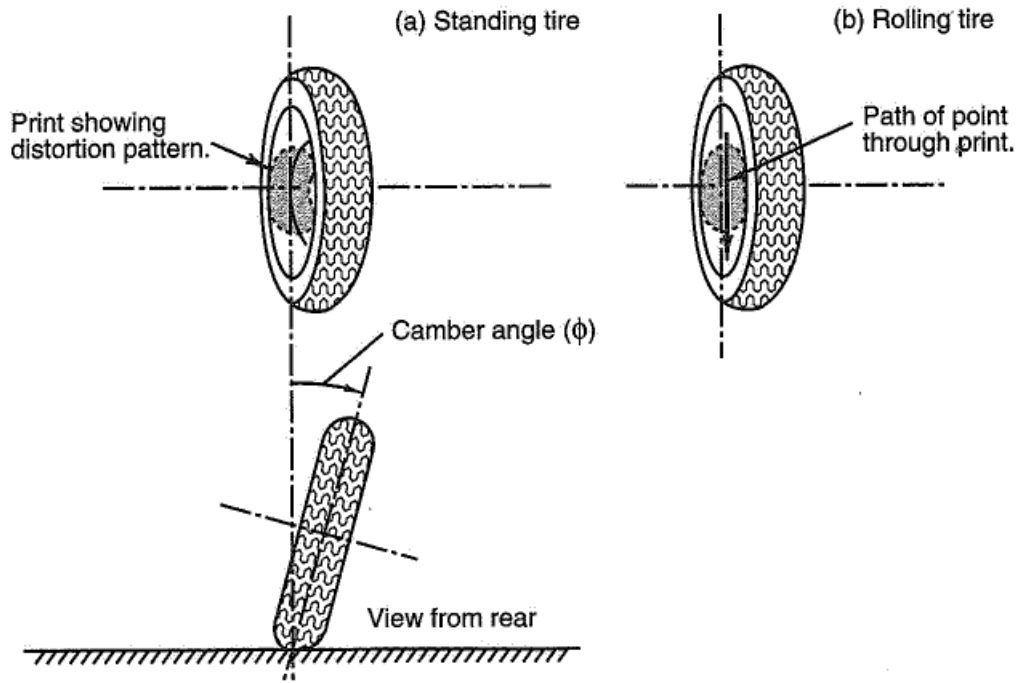


Figure 2.23. Distortion in print of a tire at a camber angle.[3]

For radial tires the camber stiffness may be quite small since the lateral stiffness of the belt and the flexibility of the radial cord sidewall inhibit the kind of print distortion that gives rise to camber thrust. Such camber thrust as is generated may be due to distortion in the tread pattern or sidewall effects. For wide Street radial tires the camber forces tend to fall off at camber angles above 5° . For racing tires, the maximum force due to camber occurs at smaller angles. [3]

2.5.2. Aligning torque due to camber

Camber also affects the aligning torque. The roughly triangular distortion pattern of the print due to slip angle (the lateral forces in the print increasing toward the back) gives rise to an aligning torque which is “stabilizing” in the linear range. However, aligning torques due to camber are generally quite small because of the fore and aft symmetry of the print distortion. In practice small “destabilizing” aligning torques usually result when lateral force is generated by camber alone, i.e., the aligning torque due to camber tends to increase the slip angle. In

combination with slip angle, both effects tend to cancel and this may require an increase in mechanical trail to produce the desired cornering effect. [3]

2.5.3. Camber roll-off and peak lateral force

In the linear range, camber thrust and lateral force due to slip angle are generally viewed as separate effects and are additive. This simply moves the cornering curve (lateral force vs slip angle) up or down parallel to itself depending on which way the wheel is cambered. As the linear range is exceeded, the additive camber effect decreases or is said to “roll-off” (figure 2.24). Measurements made on a narrow bias-ply tire demonstrate that the peak of the cornering curve can move up when the tire is tilted into the turn while the peak generally moves down if the tire is tilted outward. Figure 2.25 shows this effect for round cross-section, slick bias-ply tires. [3]

Road-induced distortion due to slip angle is a maximum toward the rear of the print where local vertical forces are low; the largest distortion in the case of camber is near the center of the print where the local vertical forces are high. Slippage in the highly loaded center of the print is less likely than at the lightly loaded end of the print. This may help explain why higher lateral force is achieved when “aiding” camber is added to the slip angle. Figure 2.26 is a plot of the maximum lateral force obtained on a Goodyear Eagle tire for various loads and cambers. As in figure 2.25 the slip angle varies along each curve (the value with the max lateral force at each point is used). The highest lateral force values are all at some nonzero camber angle. The best camber angle increases with load, except for the light load where the data is probably less reliable. At 1200 lb. Load, a 6% increase in max lateral force is obtained at -6° of camber. [3]

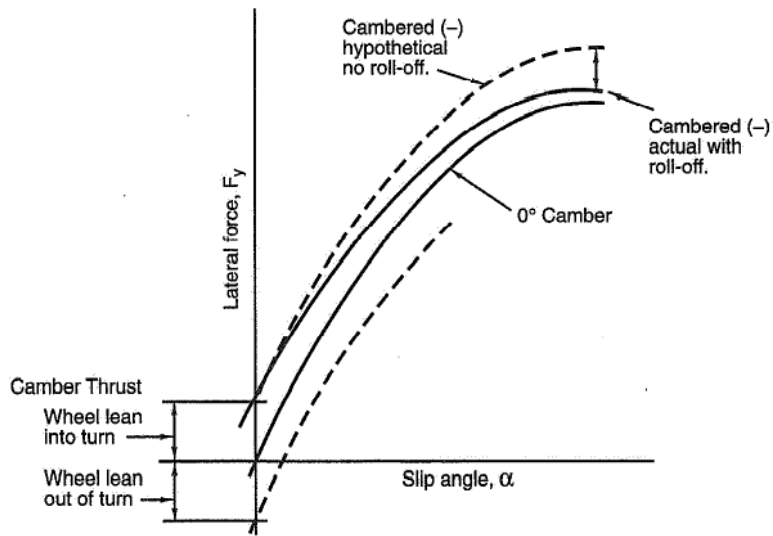
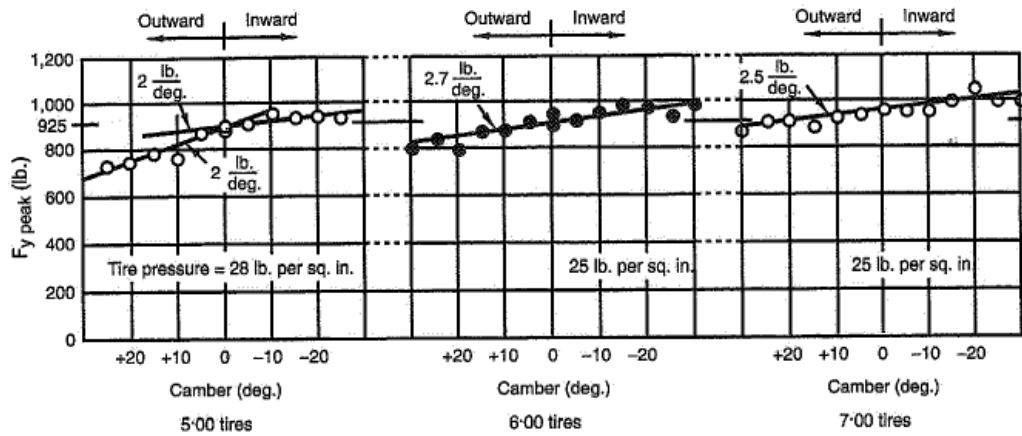


Figure 2.24. Camber thrust and camber roll-off at constant load.[3]



Notes: The slip angle at each point on these curves is that which gave maximum lateral force. 925 lbs. load for all tires

Figure 2.25. Peak side force v camber.[3]

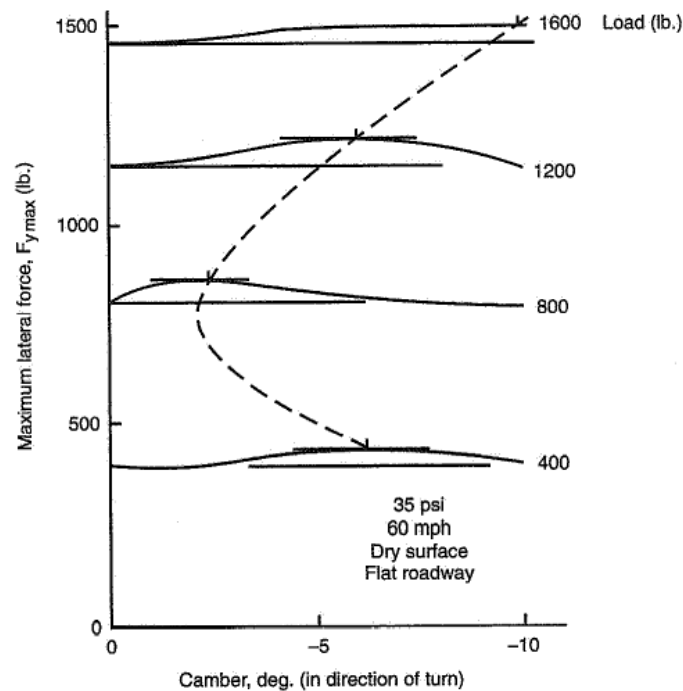


Figure 2.26. Peak lateral force v camber. PS225/70R15 tire.[3]

2.5.4. Typical cambered tire data

The next three figures are useful to understand the the general effect of camber on lateral force over the full range of load and slip angle. The tire was tested at 35 psi pressure at 60mph under dry conditions. Figure 2.27 is at 0° camber. The curves are labeled in terms of steer angle instead of slip angle; steering the wheels to the right is positive (+). Figure 2.27 indicates that the lateral force-producing capability of the non-cambered tire is about the same for right and left steer, hence the tire behavior is nearly symmetrical. [3]

Figures 2.28 and 2.29 are for the wheel tilted at 5° and 10° to the right, respectively. In both instances the curves are asymmetrical. When the wheel is tilted to the right the lateral force produced by the right steer is greater than with left steer. The camber has the effect of rotating the set of curves in an counterclockwise direction around the origin. For these relatively wide tyres, -5° of camber yields most of the gain in lateral force from camber. [3]

With the exception of oval tracks (where both wheels on an axle may be tilted in the same direction), it is common to use negative camber. With modern wide

radial tires, the camber is restrained to, say -5° for reasons of temperature, wear and performance. [3]

2.5.5. Camber optimization

Curves such as those of figures 2.27, 2.28 and 2.29, or normalized versions, are useful for adjusting the camber to achieve maximum lateral force on a wheel at a known load and slip angle condition. Figure 2.30 is a plot of normalized lateral force vs. load for three angles all at 7° of steer angle in a right-hand turn. This steer angle was chosen as close to the peak for the three camber angles. Using this limited data, we can approximate the desirable camber for the outside front wheel for a range of vertical loads to maximize F_y/F_z . [3]

All of the curves slope downward to the right, as was noted for vertical wheels in the earlier section on tire load sensitivity. The introduction of camber changes both the coefficient level and the load sensitivity (slope of curve). In this example, 5° of negative camber appears desirable at any load likely to be seen on the outside wheel. To truly find the optimum camber angle, data would be required at other intermediate camber angles near -5° . [3]

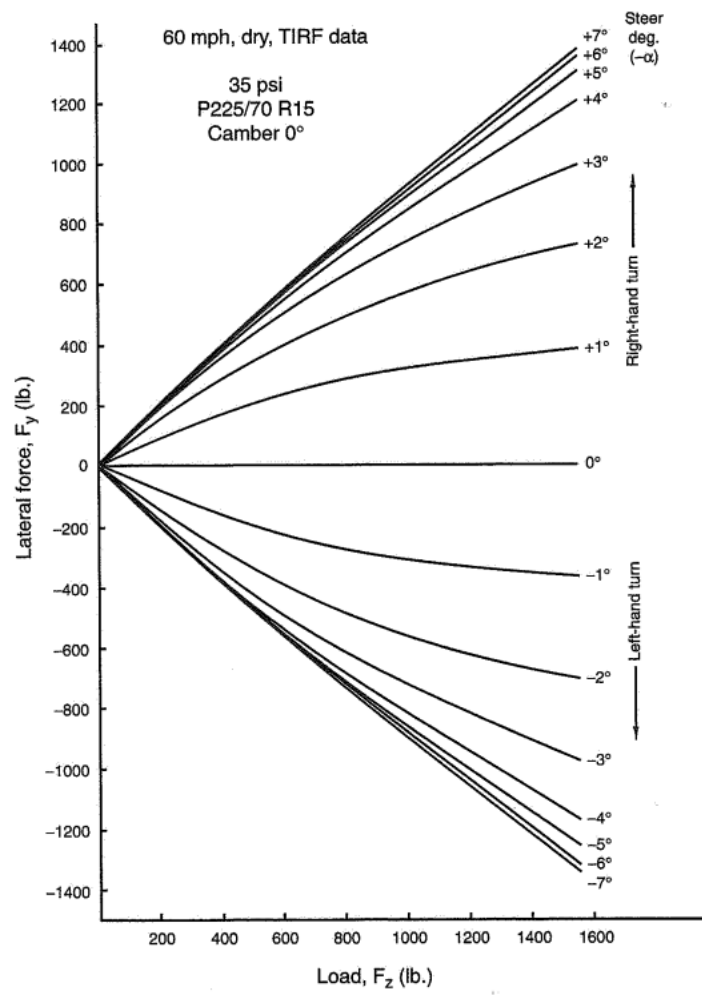


Figure 2.27. Lateral force at zero camber.[3]

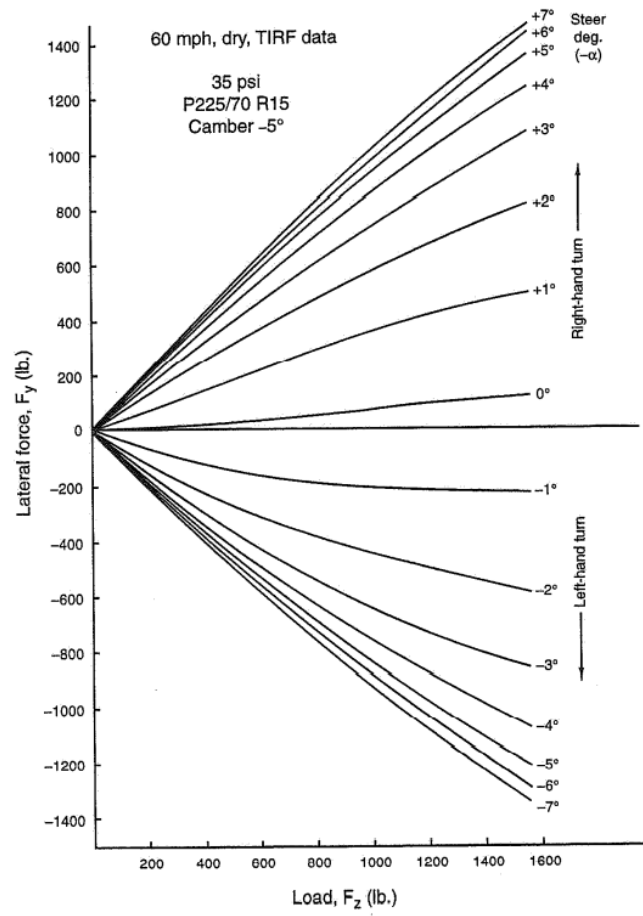


Figure 2.28. Lateral force at -5 degrees of camber, lean to right for LH front wheel.[3]

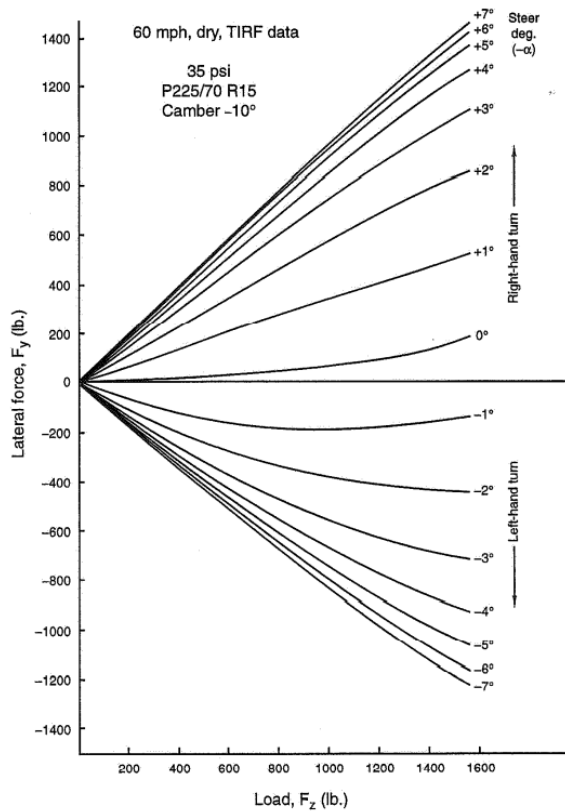


Figure 2.29. Lateral force at -10 degrees of camber, lean to right for LH front wheel.[3]

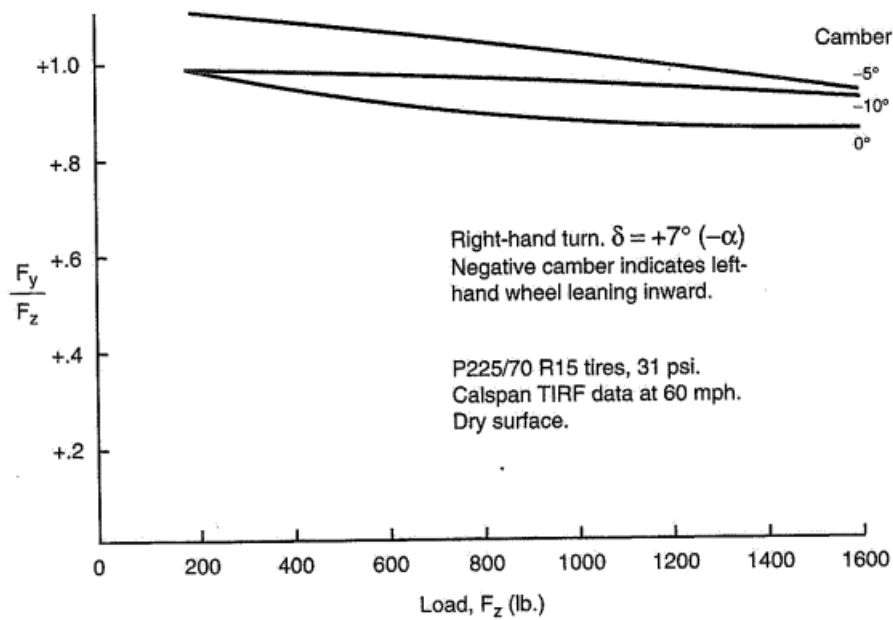


Figure 2.30. Load sensitivity for several camber angles.[3]

2.6. Other tyre effects

2.6.1. Tire pressure

This is one of the most common and easiest changes made in setting up a race car. The range of permissible pressures are usually set by the tire manufacturers.

Tyre performance at low slip angles and modest side forces is described by the cornering stiffness or the slope of the lateral force vs. slip angle curve of figure 2.8. This slope is a measure of the tire's elastic properties, if the tire pressure is raised, the carcass will get stiffer and the cornering stiffness will increase. For a given small slip angle, an increase in pressure will give an increase in lateral force. By varying pressures front to rear, modest changes in the under/oversteer balance of the vehicle can be achieved. Tyre performance at the peak of the curve (figure 2.8) depends on the effective friction coefficient. Despite the fact that grip is not very well understood, in general lowering the contact pressure between the rubber tread and the road raises the effective friction coefficient. Lowering the air pressure increases the size of the footprint and lowers the contact pressure in the print. However, too low air pressures create localized high pressure areas at the sides of the print due to the sidewall stiffness. The general idea is to set the air pressure such that the center tread and the edges of the tread carry a proportional share of the load. [3]

Aligning torque is also affected by pressure. As the pressure is decreased, the print becomes longer and the center of lateral force moves rearward. When a tire loses air, steering effort for a given lateral acceleration turn is increased because of the decrease in cornering stiffness and the increase in aligning torque. [3]

The drag will be decreased with an increase in tire pressure. There are two mechanisms of tire drag. The rolling resistance is a measure of energy lost in deforming the tire, with higher pressure, distortion and print size are reduced. The tire induced drag (drag resulting from cornering) will be reduced for a given lateral force (below the limit) if the pressure is increased because the required slip angle is lower. [3]

Ride, or handling, is adversely affected by an increase in pressure, the tire spring rate increases and the tire contributes less compliance to the vertical springing of the vehicle. The vertical tire spring rate is critical in race cars with stiff springing

(i.e., ground effects cars) because the tire is a significant part of the total spring rate. [3]

Hence, the optimal tire pressure is a compromise between the above factors.

2.6.2. Tire temperature

Tire temperature affects both the force producing capability of the tire and also the life of the tire. The first question that must be considered regarding tire temperature is, "What temperature"? Working from the outside in, it is possible to measure:

- Different temperatures on the surface of the tire tread or the surface of the sidewall.
- Temperatures inside the tread or carcass with a needle probe.
- The contained air temperature (CAT) of the tire, which is an average temperature of the inside of the tire and rim.

The tire carcass is an elastic device that is partly responsible for the cornering stiffness. A change in temperature will change the modulus of elasticity of the rubber and affect the cornering stiffness. [3]

Tire pressure and temperature are interrelated, lower initial pressure results in more rubber distortion and higher temperature, which in turn results in higher "hot" pressure. [3]

Modern race tire tread compounds have an optimum temperature for maximum grip. If too cold, the tires are very slippery; if too hot the tread rubber will "melt"; in between is the correct temperature for operation. [3]

2.7. Degradation phenomena

Depending on the temperature of the tyre, different degradation phenomena can appear and affect the structure of the rubber. As mentioned before, there is a range of temperatures in which the tyre is designed to operate. In this range, wear will take place, but in a manageable way.

2.7.1. Graining

At the beginning of a driving session, it is common for tires to be cold on both the inside and outside [8]. If the tire is subjected to excessive loads at this stage, the brittle properties of cold rubber can cause deformation beyond the recovery point. These imperfections are called graining and cause pieces of rubber to break off or roll off the tire, as shown in Figure 2.31. [9]



Figure 2.31. Graining. [9]

2.7.2. Wear

As the rubber compound reaches its operating temperature, flexibility increases. When high forces and slip speeds are combined, the rubber experiences extreme stress and localized temperature peaks around the asperities in the road. This phenomenon is known as flash temperature, which causes the composition of the rubber to change, creating a thin layer with poor mechanical properties that can be easily eroded, similar to graining. It is assumed that the rate of wear is proportional to the slip power of the contact patch.[9]

2.7.3. Blistering

When the tire overheats, the rubber goes into an inversion state, meaning it reverts to its naturally soft state rather than vulcanizing to a stiffer state. If this only happens on the outside, usually when a wheel locks up when braking, the soft rubber is scrapped off. While this is not critical, it causes instability. [9]

2.8. Speed effect

Tire performance varies with speed. Tire force capability drops off as speed is increased. This drop-off may be gradual or abrupt in a narrow range of speeds.

Laboratory testing for speed effect requires separating speed effects from temperature effects. As the tire speed is increased at constant slip angle, the temperature increases during the test. [3]

2.9. Friction circle and ellipse

Figures 2.32 and 2.33 illustrate the lateral force F_y and the longitudinal force F_x as functions of slip angle and traction/braking slip ratio. In this section, a single diagram is developed which combines the longitudinal and lateral forces. This diagram is composed of friction ellipses and the friction circle. The friction circle represents the force-producing limit of the tyre for a given set of operating conditions (load, surface, temperature, etc.). This diagram for a single tire is similar to the friction circle diagram for the whole vehicle. [3]

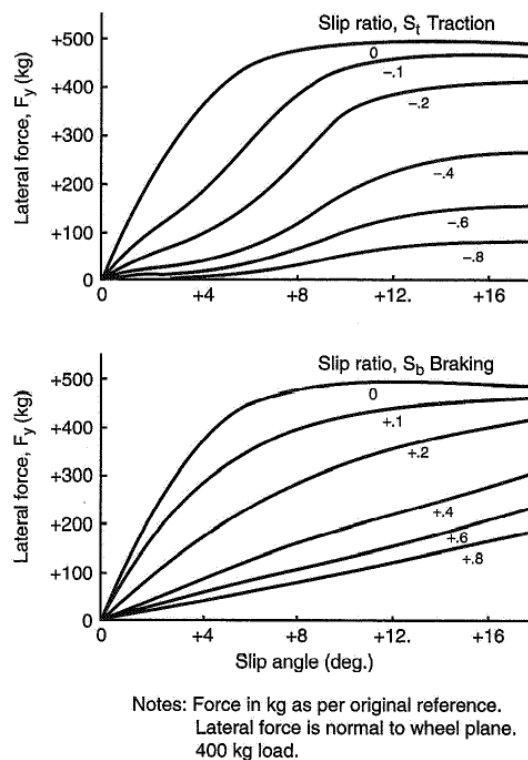


Figure 2.32. Lateral force v slip angle. [3]

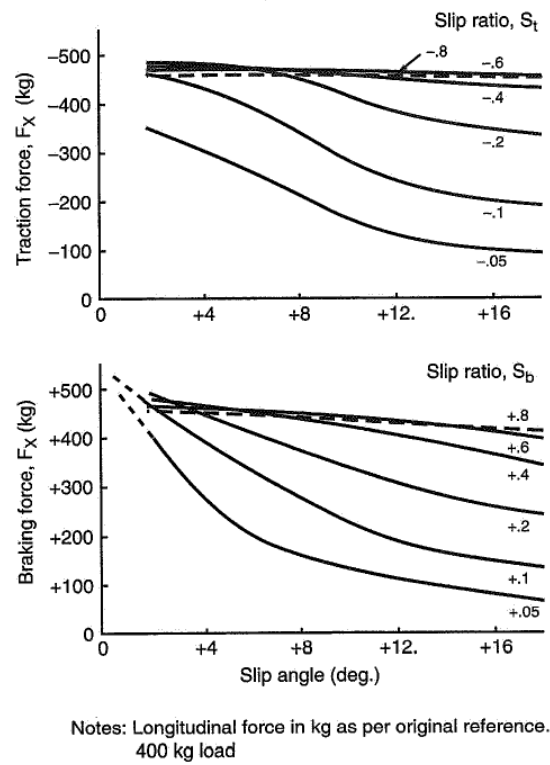


Figure 2.33. Tractive and braking force v slip angle.[3]

Figure 2.34 is a plot of lateral force vs. longitudinal force for a series of slip angles and slip ratios. Only right half of the circle diagram is shown. Lateral force is measured perpendicular to the wheel plane and longitudinal force is measured in the wheel plane. [3]

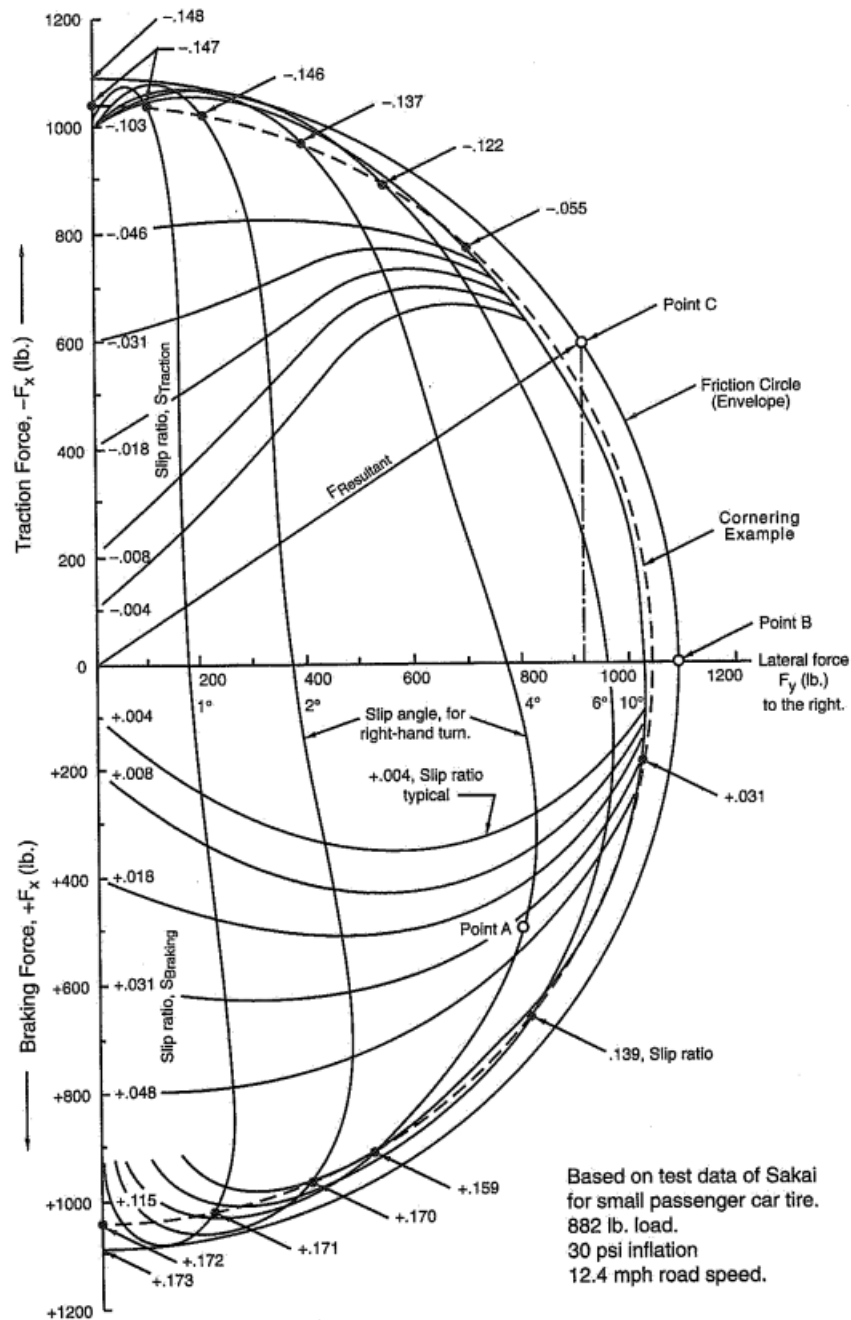


Figure 2.34. Friction circle diagram RH turn.[3]

2.9.1. General description of the diagram

At the origin the tyre is free-rolling straight ahead with no longitudinal force (only a small rolling resistance). The slip angle and lateral force are zero. Moving to the right along the horizontal axis increases lateral force and the corresponding slip angle lines are notes; the conventional lateral force vs. slip angle curve could be taken from the data on the horizontal axis.[3]

The vertical axis represents longitudinal force and various slip ratios are marked along the scale. Note that + is down and braking while traction is up and -. Points on the vertical axis could be used to develop the curves of slip ratio vs. longitudinal force. [3]

The interesting part of the diagram is off axis, where combined traction/braking and lateral force are represented. For example, point A on the figure represents the force that is generated by 4° of slip angle and a slip ratio of about +0.036. This can be read off the force scales on the axes as 800 lb. of lateral force and 500 lb. of longitudinal force (braking). [3]

In general, a circle is a close approximation to the boundary of the diagram, this “friction circle” represents the maximum force that the tire can generate under these operating conditions. In racing, the tire is operated at or near the limit of the friction circle. At point B for example, the tire is giving pure lateral force at its best slip angle (13° slip angle gives this peak, although this curve is not drawn on the figure) of 1100 lb. If 600 lb. of tractive force is applied, the available lateral force drops to about 920 lb. at point C. The combined force, the vector sum of the lateral and longitudinal forces, is still 1100 lb. as shown by the arrow labelled $F_{resultant}$. From the right triangle formed by the components of $F_{resultant}$, the Pythagorean Theorem can be used to calculate the length (force) from the origin to point C. [3]

$$F_{Resultant}^2 = F_x^2 + F_y^2$$

$$F_{Resultant}^2 = \sqrt{600^2 + 920^2}$$

$$F_{Resultant} = 1100lb.$$

2.9.2. Cornering example

As a preliminary to the discussion of the g-g diagram, the friction circle diagram (Figure 2.34) will be used to describe racing cornering maneuver of a very simple vehicle. The friction diagram is drawn for a simple tire, so the car is simplified. Due to this, no load transferences are possible, so the load is constant at the 882 lb. for the following example. [3]

The limit for this tyre is about 1100 lb. resultant force. The corner is taken at $F_{resultant} = 1045$ lb. or 95% of the limit force. This is shown on the figure by a dashed line labelled “Cornering example”.

If the turn is negotiated at 12.4 mph (the speed at which the tire data was obtained) the resulting slip velocity at 1045 lb. force would be 2.15 mph. Instead we choose to ratio the speed up, for example, at 100 mph the slip velocity increases by the ratio of the new to the original speed or $100/12.4 = 8.06$. The new slip velocity is $2.15 * 8.06 = 17.34$ mph. [3]

Before turn entry, the tire decelerates at 1045 lb. which translates to a slip ratio of +0.172. As the vehicle is turned in, lateral force builds up while braking force must reduce to maintain constant resultant force. The black dots at the intersections of the slip angle curves and the dotted 95% circle are labelled with the slip ratio at each point. By the time 4° slip angle is reached (the curve which also has point A on it), the slip ratio is +0.159. At the limit of cornering, the operating point is on the horizontal axis and the slip ratio is essentially zero. [3]

As power is applied on turn exit, the slip ratio goes negative and the slip angle must decrease correspondingly. By the time the slip angle is down to 6° the slip ratio can be -0.122. At the end of the turn, the vehicle can accelerate at a slip ratio of -0.147 without any lateral force production. The resultant force on the vehicle has remained at a constant 1045 lb., only the direction of the force vector has changed. [3]

Figure 2.35 shows the control that must be provided by the driver when traversing the example corner. Here, turn entry begins at the top of the figure with the driver applying just enough brakes to give the slip ratio of 0.172 (braking slip ratio is positive). As turn in progresses, the driver uses the steering and brakes to move along the curve. We have to keep in mind that slip ratio is a function of brake torque and also steer angle, while slip angle is likewise a function of steering and braking. [3]

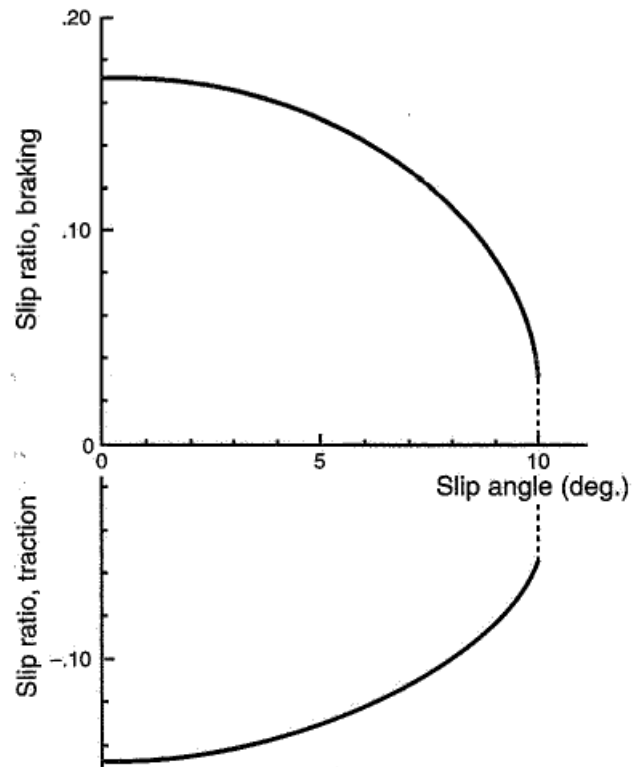


Figure 2.35. Slip ratio v slip angle for cornering example.[3]

3. Wheel loads

3.1.1. Assumptions used in this chapter

In a real car, wheel loads are constantly changing. A number of operational and simplifying assumptions are made. First, we will be dealing only with steady state operating conditions, that is, smooth roadway, constant speed cornering, constant longitudinal acceleration, constant grade, etc.

The vehicle is also in steady state, all the variables discussed are in equilibrium and transient behavior, sprung and unsprung mass dynamics are neglected.

Second, we assume that all of the basic car data used in the calculations are linear and that basic dimensional data are constant.

Third, the assumption of linearity implies that the principle of superposition is valid, this is to say, the total of a series of effects considered concurrently is identical to the sum of the individual effects considered individually.

Finally, we have to assume that the chassis is rigid. [3]

3.1.2. Center of gravity location

The location of the center of gravity (CG) is one of the most important parameters of performance because tire cornering force capability is very dependent on the vertical

load applied to the tire. Most chassis changes that are made to improve handling performance of race cars have an effect on wheel loads through changes on the position of CG or changes that affect the weight distribution during cornering. [3]

3.1.3. Total vehicle horizontal (x and y) location of the CG

In the general case in which front and rear tracks are not equal, the CG will not be necessarily on the longitudinal centerline. The centerline of the car is defined as the line which connects the center of the front and rear tracks.

To start calculating the CG, we have to settle the vehicle at ride height on a level surface, and then roll the vehicle carefully onto four individual scales that have

been leveled. We need to obtain the weight of the four wheels individually as well, and, at the same time, record the loading condition (with or without driver, amount of fuel, etc.). Measure the front and rear track width and the wheelbase is another important step for the data requirement. [3]

The total weight of the vehicle is the sum of the four individual wheel weights measured.

$$W_1 + W_2 + W_3 + W_4 = W = \text{Total weight of the vehicle}$$

The first calculation step is to take moments about the rear axle

$$b = \frac{W_F * l}{W}$$

And

$$a = l - b$$

This establishes the fore and aft location of the CG.

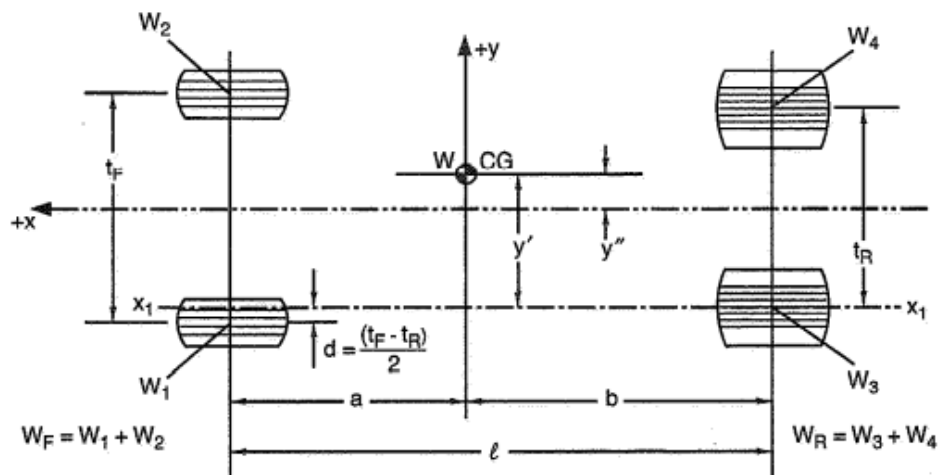


Figure 3.1. Horizontal location of the CG. [3]

Next, we can take moments about the x_1 - x_1 axis (a line parallel to the centerline of the car through the center of the left rear tire)

$$y' = \frac{W_2}{W} (t_F - d) - \frac{W_1}{W} (d) + \frac{W_4 t_R}{W}$$

This can be solved for y'' (since $y'' = (y' - t_r / 2)$) to give the lateral shift of the CG from the x axis (centerline):

$$y'' = \frac{W_2}{W} (t_F - d) - \frac{W_1}{W} (d) + \frac{W_4 t_R}{W} - \frac{t_R}{2}$$

Y'' as shown in the figure has a positive value. A negative value would represent a shift of the CG to the left as is generally the case for oval track cars. [3]

These equations apply to the general case. If $t_F = t_R = t$, the expression reduces to:

$$y'' = \frac{W_2 + W_4}{W} t - \frac{t}{2}$$

To conclude, if the front and rear tracks are equal and the CG is on the centerline, then $W_2 + W_4 = W/2$, and no lateral offset exists. [3]

3.1.4. Total vehicle vertical location of the CG

To determine the height of the CG, one method is to elevate the rear axle so that the front to rear wheel centerlines are at an angle θ with the horizontal. An example is shown in figure 3.2.

As test conditions, the suspension motion must be locked up, a way to do this is replacing the shock absorbers with rigid links that will keep the car at ride height. [3]

The front wheels are chocked on scales, and are the the only thing that avoid the car rolling, so they must be secure. Any load that could shift must be secured (fuel tank should be empty).

We also need the following data:

W Total weight of the vehicle

W_F Weight of the front wheels with rear elevated

b Horizontal distance from rear axle to CG

l Wheelbase

$$h_1 = \frac{W_F l - W_b}{W \tan \theta}$$

h_1 is the height of the CG above the line connecting the wheel centers which are at height R_l . If the R_l is the same front and rear the CG height above the ground is [3]

$$h = R_l + h_1 \quad (1.26)$$

If R_l is different front and rear, the height of the line connecting the wheel centers at the CG location must be found by [3]

$$R_{L\ CG} = R_{LF} \left(\frac{b}{l} \right) + R_{LR} \left(\frac{a}{l} \right)$$

If R_l is different front and rear the CG height is [3]

$$h = R_{L\ CG} + h_1 \quad (1.28)$$

3.1.5. Sprung mass CG location

The sprung mass CG location can be calculated if we also know the weights of the unsprung masses. Unsprung mass includes wheels, tires, knuckles, brakes, and approximately $\frac{1}{2}$ of all the connecting components. For a solid axle, the whole axle assembly is unsprung. Figure 3.3 shows a plan view of a car in the most general case. The total vehicle CG, W is located longitudinally by a and b and laterally by y' and y'' . The individual unsprung weights are located at the center of each wheel. The weight of the sprung mass is the total vehicle weight minus the unsprung weight. [3]

$$W_S = W - W_{U1} - W_{U2} - W_{U3} - W_{U4} = \textit{Sprung Weight}$$

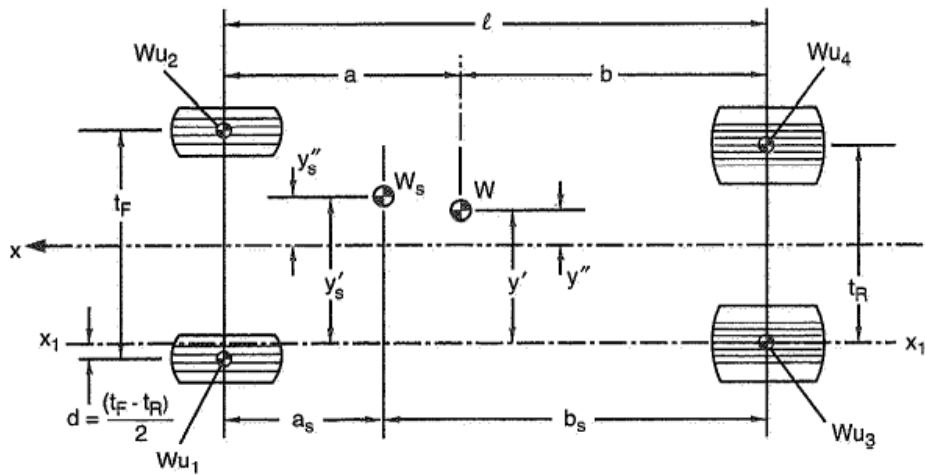


Figure 3.3. Lateral and longitudinal sprung mass CG location.[3]

The total front and rear unsprung weights are

$$W_{UF} = W_{U1} + W_{U2} = \text{Front Unsprung Weight}$$

$$W_{UR} = W_{U3} + W_{U4} = \text{Rear Unsprung Weight}$$

The longitudinal location of the sprung mass CG is found by taking moments about the rear axle:

$$b_s = \frac{Wb - W_{UF}l}{W_s}$$

And

$$a_s = l - b_s$$

The lateral location of the sprung mass center of gravity is found by taking moments about the $x_1 - x_1$ axis:

$$y'_s = \frac{W}{W_s} y' - \frac{W_{U4}}{W_s} t_R - \frac{W_{U2}}{W_s} (t_F - d) + \frac{W_{U1}}{W_s} d$$

And

$$y''_s = y'_s - \frac{t_R}{2}$$

If the front unsprung weights are the same either side ($W_{U1} = W_{U2} = W_{UF}/2$) and the rear unsprung weights are also the same either side ($W_{U3} = W_{U4} = W_{UR}/2$), then the equation for the lateral position reduces to

$$y'_s = \frac{W}{W_s} y' - \frac{W_{UR}}{2W_s} t_R - \frac{W_{UF}}{2W_s} t_F + \frac{W_{UF}}{W_s} d$$

Furthermore, if the front and rear tracks are equal ($t_F = t_R = t$ and $d = 0$), the expression becomes [3]

$$y'_s = \frac{W}{W_s} y' - \frac{W_{UR}}{2W_s} t - \frac{W_{UF}}{2W_s} t$$

The sprung mass CG height is found by viewing the x-z plane as shown in figure 3.4. Data required are the total vehicle CG height, h , and the front and rear unsprung weights, W_{UF} and W_{UR} (total for both sides), and their heights above the ground (a good first approximation is the wheel center height). The sprung mass center of gravity height, h_s , is found by taking moments about the ground: [3]

$$h_s = \frac{W}{W_s} h - \frac{W_{UF}}{W_s} R_{LF} - \frac{W_{UR}}{W_s} R_{LR}$$

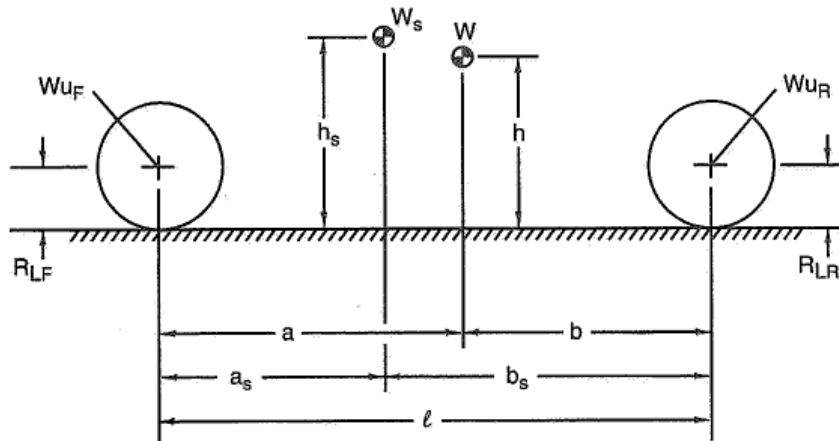


Figure 3.4. Sprung mass CG height.[3]

3.2. Chassis stiffness

Stiffness is resistance to bending or flexing, torsional stiffness is resistance to twisting. A performance vehicle must have an adequate chassis torsional stiffness around the x-axis. Think of the chassis as a large spring connecting the front and rear suspensions: if the chassis torsional spring is weak, attempts to control the lateral load transfer distribution will be confusing. This is because a flexible

chassis adds another spring to an already complex system. Predictable handling can best be achieved if the chassis is stiff enough to be safely ignored. [3]

Further arguments for a torsionally stiff chassis are

- It is not practical to dump structural twisting in the chassis with current technology.
- A chassis that flexes may be prone to fatigue, and further “softening” with use, resulting in failure.
- Suspension compliances may be increased or decreased by bending or twisting of the chassis. [3]

3.2.1. How torsionally stiff?

Typical unibody sedans are in the range of 4000 to 10000 lb.-ft./deg. Current small formula cars may be 3000 lb.-ft./deg. And up. Again, the reason for torsional stiffness is to provide a rigid platform for the suspension, to allow the lateral loads to be distributed front to rear in proportion to the roll stiffnesses of the suspension. [3]

3.2.2. Design for high torsional stiffness

The type of construction and the actual design determine the torsional stiffness that can be expected from a chassis. Probably the stiffest chassis currently made are composite stressed skin (or “monocoque”). Other types that are less efficient are:

- Aluminum and steel stressed skin as seen in many purpose-built racing cars and in most modern passenger cars.
- Space frames built of tubing as seen in many production-based cars. Tubes are arranged to form triangles with the major loads applied at the intersection of tubes, otherwise the structure will work in bending which is much less efficient than tension/compression.
- Ladder frames as used in many older design cars

Probably the easiest technique for quickly getting a feel for torsional stiffness is to build a model of the proposed design. Twisting the model chassis will reveal any areas that are especially weak in torsion. [3]

3.2.3. Increasing torsional stiffness

- Add diagonals in the roll cage. This is the thing to do if the vehicle is already built and found to be too flexible.
- Remember that the engine can function as part of the chassis provided that the loads are not so high that the block is distorted.
- If tubes must be used in bending, plate reinforcements may be used at the joints to pass load more effectively from one tube to another (see figure 3.5).
- Add additional cossmembers to the chassis.

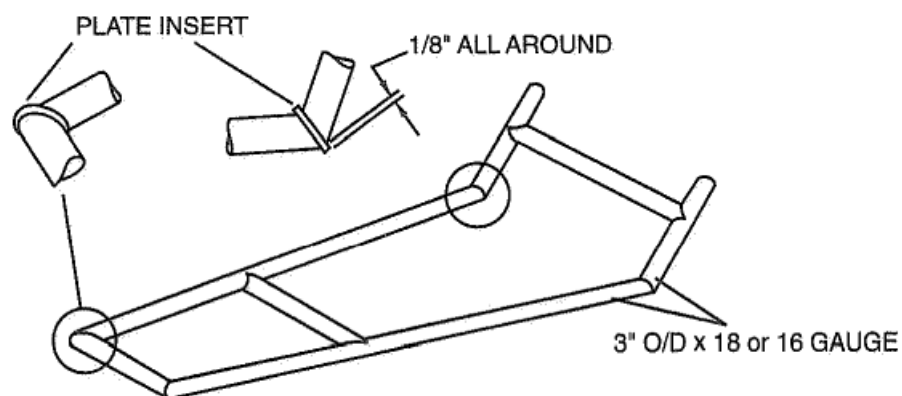


Figure 3.5. Chassis.[3]

3.3. Lateral load transfer

When cornering in a steady turn, load is transferred from the inside pair of wheels to the outside pair of wheels because the car center of gravity is above the ground.[3]

3.3.1. Total lateral load transfer

When a car is in a steady state turn and inertial reaction called centrifugal force is developed which opposes the lateral acceleration produced by tire cornering forces. If the car is compressed to a single axle, a right-hand turn situation results

as illustrated in figure 3.6. The cornering force produced by the tires, $S_L + S_R$, results in a lateral acceleration, a_y (in units ft./sec²) or A_y (in "g" units, that is, $A_y = a_y/32.2$). The inertial reaction or centrifugal force is WA_y (with sign opposite to a_y). [3]

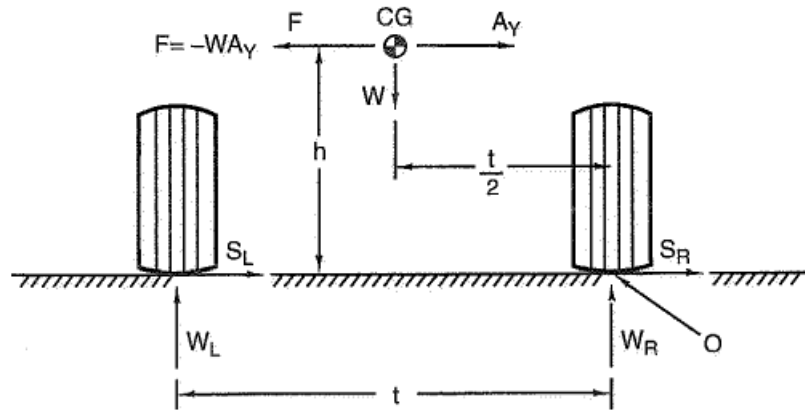


Figure 3.6. Total lateral load transfer.[3]

Taking moments about O (the right side of the tire track), we have

$$W_L t = W \left(\frac{t}{2} \right) + W A_y h$$

Or

$$W_L = \frac{W}{2} + \frac{W A_y h}{t}$$

Since the initial weight on the left-hand side of a symmetric car is $W/2$, the weight transfer due to cornering is $W_L - \frac{W}{2}$:

$$\Delta W = W_L - \frac{W}{2} = \frac{W A_y h}{t}$$

Where ΔW is the increase in left side load and decrease in right side load due to cornering. [3]

Expressed as a fraction of total weight this becomes

$$LLT = A_y h/t$$

Where

LLT = total lateral load transfer as a fraction of total weight

h = height of CG

t = track or tread width

A_y = lateral acceleration

This represents the total load transferred and does not attempt to calculate how the load is distributed on the four wheels. [3]

3.3.2. Distribution of total lateral load transfer

The information in this chapter applies only to steady turns, no consideration is given to combined lateral and longitudinal accelerations or other dynamic situations. Total lateral load transfer distribution (TLLTD) is sometimes bad called “roll couple distribution”. The problem is that roll couple distribution is only part of the mechanism that controls the distribution of loads in a steady turn. [3]

As mentioned before, load is transferred from the inside track to the outside track when cornering because of the height of CG. The distribution of this load transfer between the front and rear tracks is one of the major ways to influence under/oversteer and limit behaviour. There are two mechanisms and their effects add:

- Through the springs in proportion to relative front and rear roll stiffness.
- Direct application of loads, from the tires, to the chassis through the suspension members as determined by the heights of the front and rear roll centers.

This section describes a technique for calculating the loads on the four wheels in a steady turn once the contributions of the RCH and the roll stiffnesses are known. It is important to know the following definitions: [3]

- Roll center. That point in the transverse vertical plane through any pair of wheel centers and equidistant from them, at which lateral forces may be applied to the sprung mass without producing an angular (roll) displacement of the sprung mass.
- Neutral roll axis. The line joining the front and rear roll centers.
- Roll rate or roll stiffness. The change in the restoring couple exerted by the suspension at either pair of wheels on the sprung mass of the vehicle per

unit change in roll angle of the sprung mass about a horizontal axis. Roll rate of the whole vehicle is the sum of the separate roll rates of all vehicle suspensions.

The major assumptions that will be made in this section are:

- A horizontal lateral (transverse) load applied anywhere along the neutral roll axis (NRA) produces no roll of the sprung mass.
- The front and rear roll rates are included in the front and rear roll rate values.
- Solid axle roll relative to the ground will not enter into the calculations.
- The vehicle CG and roll centers are located on the centreline of the car.

The approach taken here will be to treat the car as a three mass system, consisting of the sprung mass (the chassis and body) and the front and rear unsprung masses (wheels, brakes, hubs, axles, and half of the mass of A-frames, springs, and shock absorbers). [3]

Figure 3.7 shows the geometry involved.

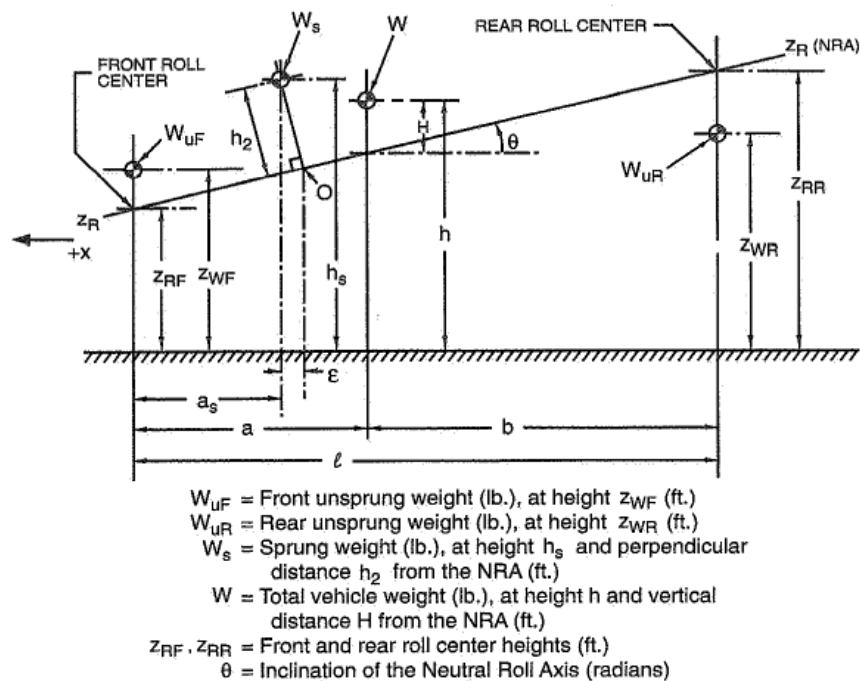


Figure 3.7. Lateral load transfer geometry.[3]

In the first step we look forward along the NRA and note that the lateral acceleration, A_Y , produces a force, $F_S = -W_S A_Y$, and, in turn, a moment about O of magnitude

$$M_S = -W_S h_2 (A_Y - \Phi)$$

Where Φ = the roll angle of the chassis.

The roll term is a gravity component which is included for completeness but can be neglected for small roll angles. Note that A_Y is positive (a RH turn), the moment M_S , on the sprung mass is negative and produces a negative roll angle, that is, the car rolls to the outside of the turn. [3]

The effects of force, F_S , at height h_S , is the same as a force, F_S , at the origin, O, on the NRA and a moment M_S (above). The force, F_S , can be split into front and rear components which will produce load transfers at the axles independent of the roll rates. The moment, M_S , produces a roll angle, Φ , measured in a plane perpendicular to the NRA whose magnitude will depend on the sum of front and rear roll rates (or roll stiffnesses), K_F and K_R ; for small θ these rates can be assumed to be around the NRA. [3]

Next, equate M_S to the roll stiffness moment and get

$$\frac{\Phi}{A_Y} = \frac{-W_S h_2}{K_F + K_R - W_S h_2} = K_\Phi$$

Which gives the roll sensitivity to lateral acceleration in radians per “g”.

Next, the moment, M_S , is split fore and aft according to their individual roll stiffnesses and the previously apportioned F_S (fore and aft) are all combined to produce individual load transfers at each axle (including the inertial reactions of W_{UF} and W_{UR}).

Using all the above we finally arrive at lateral load transfer equations for the front and rear axles that are expressed as sensitivities to A_Y . [3]

$$\frac{\Delta W_F}{A_Y} = \frac{W_S}{t_F} \left[\frac{h_2 K_{F'}}{K_F + K_R - W_S h_2} + \frac{l - a_s}{l} Z_{RF} \right] + \frac{W_{uF}}{t_F} Z_{WF}$$

Where

$$K_{F'} = K_F - (l - a_s) W_S h_2 / l$$

For the front axle, and a similar expression for the rear axle:

$$\frac{\Delta W_R}{A_Y} = \frac{W_S}{t_R} \left[\frac{h_2 K_{R'}}{K_F + K_R - W_S h_2} + \frac{a_s}{l} Z_{RR} \right] + \frac{W_{uR}}{t_R} Z_{WR}$$

Where

$$K_{R'} = K_R - a_s W_S h_2 / l$$

These equations can be simplified if one uses the total vehicle CG and its height above the NRA (H in figure 3.7). This implies a single mass system and the following replacements are made in the equations: [3]

$$W_{uF} \approx W_{uR} \approx 0$$

$$W_S \approx W$$

$$a_s \approx a$$

$$h_2 \approx H$$

$$h_s \approx h$$

$$W_{F'} \approx K_F$$

$$W_{R'} \approx K_R$$

Also, the gravity term associated with roll angle, Φ , can be neglected. If these assumptions are made, the above equations reduce to

$$\frac{\Phi}{A_Y} = \frac{-WH}{K_F + K_R} = K_\Phi$$

$$\frac{\Delta W_F}{A_Y} = \frac{W}{t_F} \left[\frac{HK_F}{K_F + K_R} + \frac{b}{l} Z_{RF} \right]$$

$$\frac{\Delta W_R}{A_Y} = \frac{W}{t_R} \left[\frac{HK_R}{K_F + K_R} + \frac{a}{l} Z_{RR} \right]$$

3.4. Longitudinal weight transfer

When a car is under positive acceleration, a_x , or negative acceleration, $-a_x$, an inertial reaction force is developed that is similar to centrifugal force in a turn. In a

turn that force is WA_y ; under longitudinal acceleration the force, F , is WA_x (with sign opposite to a_x), where A_x is a_x expressed in "g" units; that is, $A_x = a_x/32$. [3]

Figure 3.8 shows the case for driving traction.

Taking moments about O, we have

$$\Delta W_x l = hWA_x$$

And

$$\Delta W_x = \frac{h}{l} WA_x$$

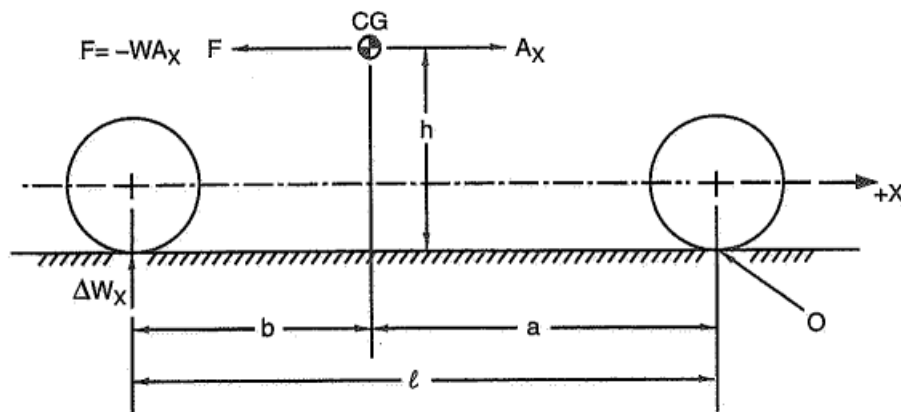


Figure 3.8. Longitudinal weight transfer - driving.[3]

Where ΔW_x is the increase in rear axle downward load (and the corresponding upward increase in the road reaction on the axle) or the corresponding decrease in front axle load. [3]

For negative accelerations (braking), ΔW_x is given by the same expression but the front axle load is increased.

In some suspension designs there is a significant change in ride height with longitudinal acceleration so it is very important to make sure that the correct dynamic CG height, h , is used. This ride height change is due to suspension geometry effects and ride/pitch effects that are variously called anti-dive, anti-lift, and anti-squat. [3]

3.5. The effects of banking

The discussion in this chapter will be limited to steady-state cornering, without longitudinal acceleration. The CG is assumed to be on the centreline of the car. In figure 3.9 is shown a car cornering at speed V and radius R , measured in a horizontal plane passing through the CG, on a banked road with bank angle α . [3]

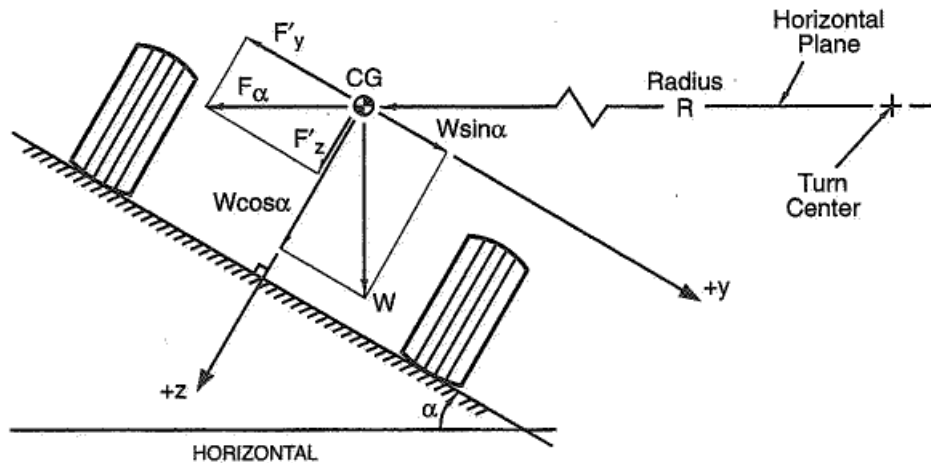


Figure 3.9. Banked RH turn, looking forward.[3]

The centrifugal force, F_α , is no different than that for a car cornering on a horizontal surface and is equal to WA_α , where

$$A_\alpha = V^2/Rg$$

Resolving this force along the y and z axes we have

$$F'_y = -F_\alpha \cos \alpha \text{ and } F'_z = -F_\alpha \sin \alpha$$

If we similarly resolve the gravitational force, W , along y and z and add the components to F'_y and F'_z we have

$$F_y = -F_\alpha \cos \alpha + W \sin \alpha \text{ (along the y axis)}$$

$$F_z = F_\alpha \sin \alpha + W \cos \alpha \text{ (along the z axis)}$$

Since $F_\alpha = WA_\alpha$ these expressions become:

$$F_y = W(-A_\alpha \cos \alpha + \sin \alpha)$$

$$F_z = W(A_\alpha \sin \alpha + \cos \alpha)$$

If $\alpha = 0$ but $A_\alpha \neq 0$ (turning in a horizontal plane), these reduce to ($\cos \alpha = 1$ and $\sin \alpha = 0$):

$$F_y = -WA_\alpha (\text{like } -WA_Y \text{ for the horizontal plane case})$$

And

$$F_z = W, \text{ as expected.}$$

From equation (2) it is seen that the effect of banking is to change the “weight” component of tire loads. If the changed weight is called W' then

$$W' = W(A_\alpha \sin \alpha + \cos \alpha)$$

Note that if $A_\alpha \sin \alpha$ is very small and $\alpha > 0$, W' will be less than W , since $\cos \alpha$ is always less than 1.0. For example if $A_\alpha = 0$ (car is stationary), $W' < W$. On the other hand, if $A_\alpha \sin \alpha$ is large, W' will be greater than W . [3]

Finally, the new front and rear weight distributions are

$$W'_F = W' \frac{b}{l}$$

And

$$W'_R = W' \frac{a}{l}$$

Note that these weight changes are symmetrical left and right and do not result in lateral load transfer. [3]

The force, F_y , of equation (1) does produce lateral load transfer and has to be added to the weight changes discussed above. However it is important to recognize that in this section we are using a single-mass approach, comparable to the simplified approach of the Lateral Load Transfer Distribution section, and not the more exact three-mass approach. If one goes through the same procedures as in the previous section one finds that the resulting expressions for lateral load transfer are similar to the simplified case of the previous section. The equations are [3]

$$\Delta W_F = (A_\alpha \cos \alpha - \sin \alpha) \left(\frac{W}{t_F} \right) * \left[\frac{HK_F}{K_F + K_R} + \frac{b}{l} Z_{RF} \right]$$

And

$$\Delta W_R = (A_\alpha \cos \alpha - \sin \alpha) \left(\frac{W}{t_R} \right) * \left[\frac{HK_R}{K_F + K_R} + \frac{a}{l} Z_{RR} \right]$$

Where ΔW_F and ΔW_r are positive for an increase in load on the outside wheels.

The difference between these expressions and the previous ones is that the gravity component is independent of A_α and prevents writing the expressions in the $\Delta W/A_\alpha$ form. Also, note that if $\alpha=0$ these expressions reduce to the previous ones (i.e., A_α becomes A_Y).

The values for ΔW given by eqs. ΔW_F and ΔW_R represent an increase in load on the outside tire only if $A_\alpha \cos \alpha > \sin \alpha$, that is, the centrifugal force component is greater than the gravity component. [3].

4. Advanced vehicle dynamics

4.1. Low and medium speed corners

We will start by describing how a vehicle would behave in an ideal situation, i.e. without taking into account effects such as the presence of slip angle because of the deformations on the tyres. Figure 4.1. shows a car going forward and intending to turn right. The direction of the front tyre is the same as the direction of the turn to be made by the car, since the directional axis is the front one, while the rear tyre's direction remains forward.

R is the turning radius, and point D is the centre of rotation of the vehicle, which is obtained from the intersection of the perpendicular to the direction of the front tyre and the perpendicular to the direction of the rear tyre. If we centre at D to the front tyre, the curve obtained is the turn of the car. In addition, r is the yaw, which is also the turn of the car, but on itself. In conclusion, turn and yaw are not the same thing, but they are equal.

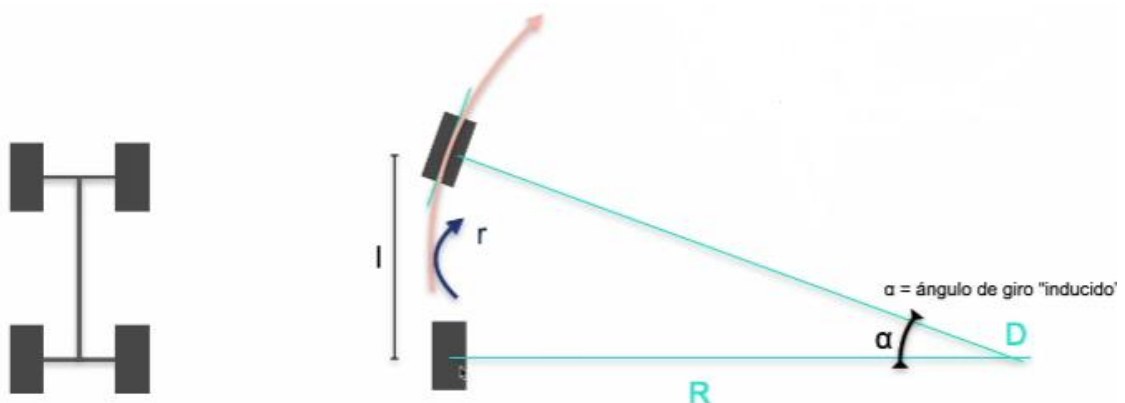


Figure 4.1. Vehicle basic scheme for a RH turn.[10]

α is the induced steering angle, and is easy to see that the greater the steering wheel angle of rotation, the greater the induced angle of rotation. l is the wheelbase.

And now we can obtain V (speed of rotation), r and α .

$$V = R * \omega$$

$$r = \frac{V}{R}$$

$$\sin \alpha = \frac{l}{R}$$

The yaw rate (r) is very important in motorsport and in the automotive field because we can determine how fast the car is turning on itself, i.e. if the yaw rate is cancelled, how fast it would pivot on itself.

Figure 4.2 shows a basic scheme for representing a right-hand curve at medium or low speed, where the front tyre is the one that is steered. As mentioned in previous chapters, we must take into account the presence of the slip angle, which makes the actual trajectory of the tyre different from the ideal trajectory of the vehicle during the curve. The real tyre directions are represented by V_f and V_r , (front and rear, respectively) and the slip angle by α_f and α_r and l is the wheelbase.

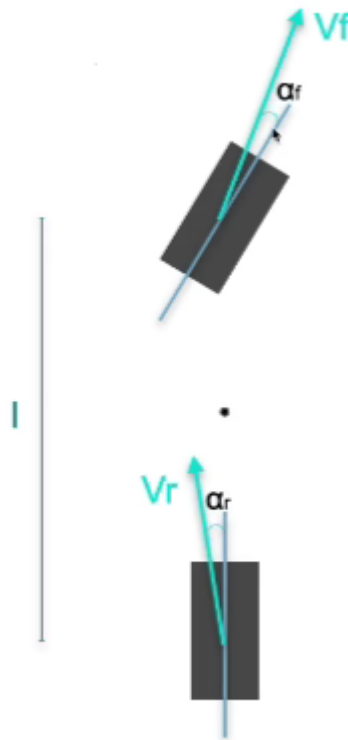


Figure 4.2. Basic scheme RH turn.[10]

Now, unlike in the previous case where the slip angle on the tyres was not taken into account, when we draw the perpendiculars to the real tyre paths, we can see that point D has been displaced, as shown in figure 4.3. R is the turning radius, and it goes from D to the centre of gravity of the vehicle (in this case it is assumed to be at the centre). V is the actual velocity of the car, and is perpendicular to the line R. V is divided into its vector component V_x , which is the tangent speed and V_y .

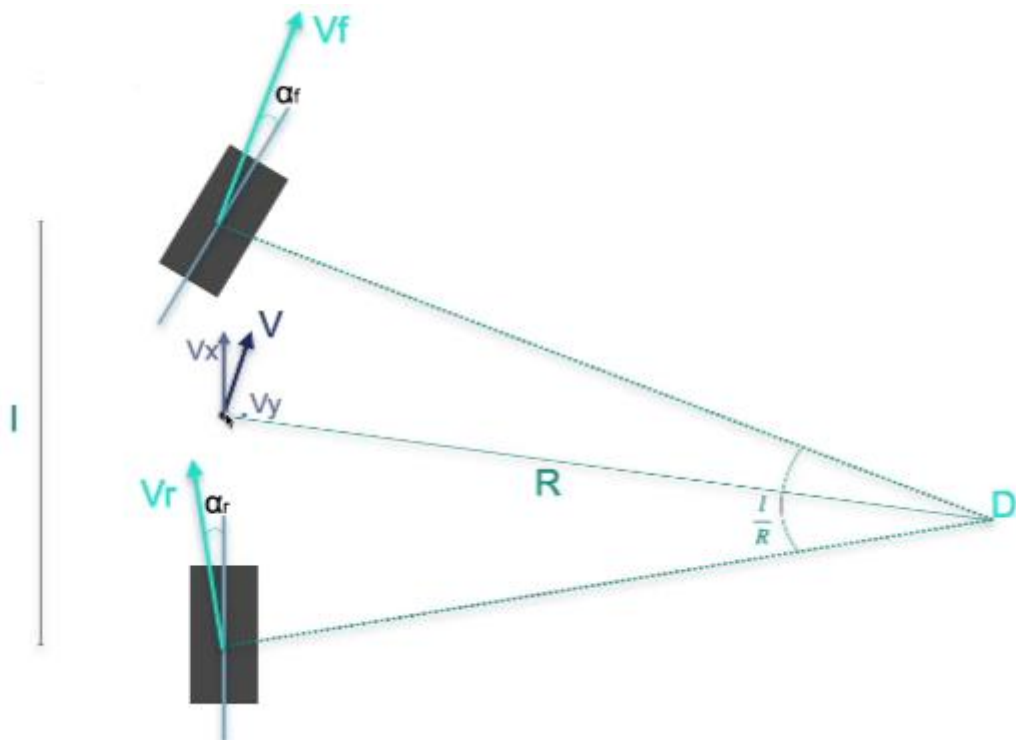


Figure 4.3. Low speed corner scheme.[10]

Figure 4.4 shows that by separating by a vertical line the ideal front tyre trajectory and α_r , we have the steering angle δ on the right and the rear slip angle α_r on the left.

However, in Figure 4.5, if instead of a vertical line, we were to place the actual trajectory followed by the front tyre, the angle on the left would be $\frac{l}{R}$ and on the right α_f .

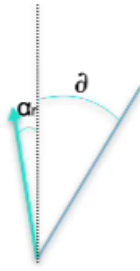


Figure 4.4. Steering angle and slip angle.[10]

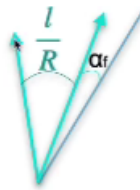


Figure 4.5. trigonometric relationship for slip angle and geometry of the car.[10]

From the above, the following equation can be derived:

$$\alpha_r + \delta = \alpha_f + \frac{l}{R}$$

And what happens if the speed is increased? The slip angle on the front and rear tyres will be higher. By drawing perpendiculars to the actual tyre trajectories, point D will now be above the centre of gravity, so the speed, V , will take on an "outward" direction as illustrated in figure 4.6.

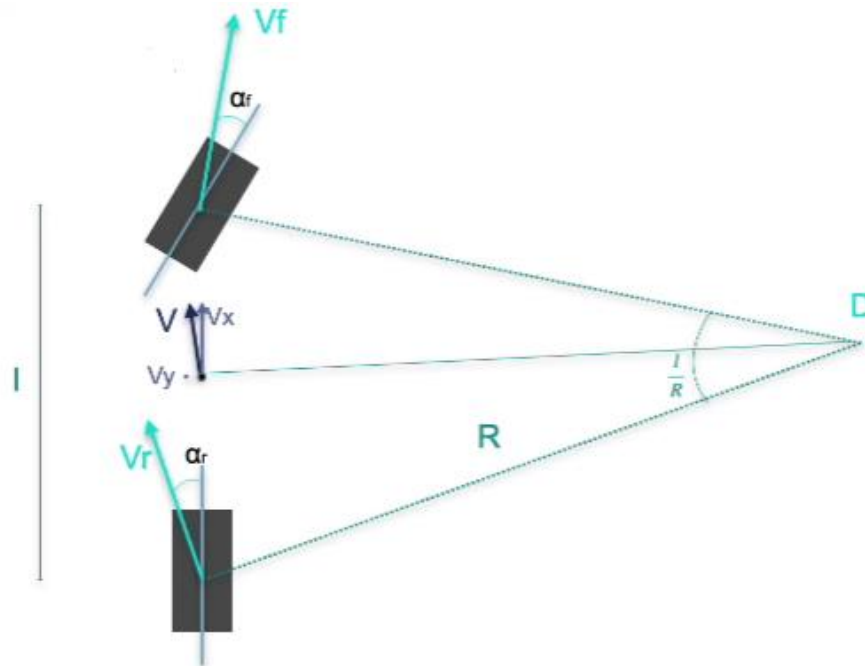


Figure 4.6. Medium speed corner.[10]

4.2. Influence of the CG

Figure 4.6 shows a basic scheme for a medium speed corner for a vehicle with his CG or center of masses centred at $l/2$. However, this representation is not correct in most cases, as the centre of mass of the vehicle will rarely be right in the centre. Let us see in figure 4.7 what happens with a forward centre of mass. The only thing that changes in this diagram is the position of the centre of mass, there is still slip angle at the front and rear tyres and point D is obtained from the intersection of the straight lines perpendicular to the actual direction of the front and rear tyres. However, the R now, which is the line from point D to the centre of mass of the vehicle, causes the tangential velocity component and the velocity of the car to change with respect to the previous example, and it can result, as in this case (figure 4.7), that the car speed coincides with the tangential velocity.

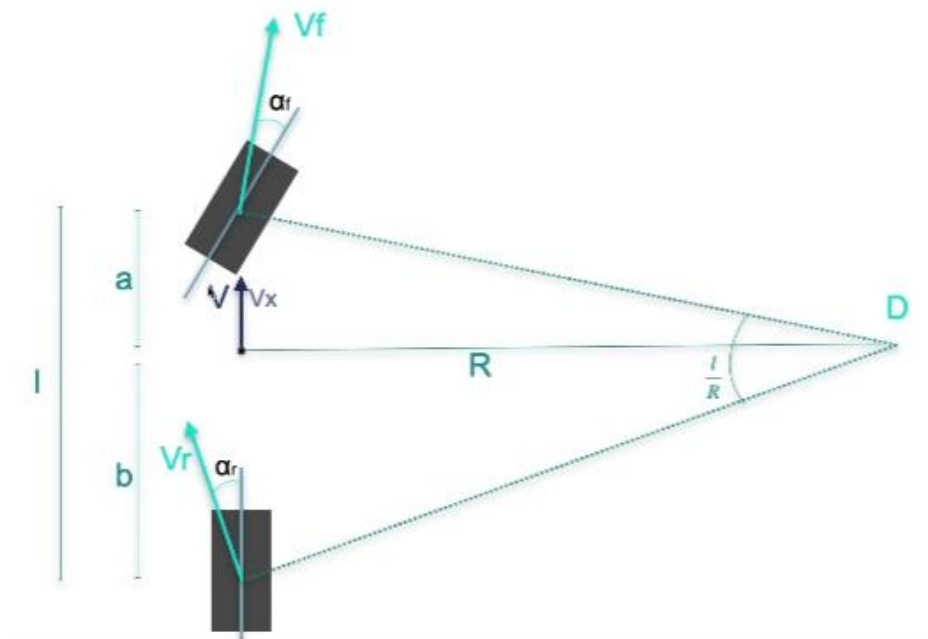


Figure 4.7. Example for a forwarded CG wher V_x is coincident with V . [10]
 Therefore, what has been achieved by moving the centre of gravity forward has been a change in the speed of the car and its tangential component.

In addition, it should be noted that new components appear from the centre of the tyres (Figure 4.8). These are, on the one hand V_{fx} , V_{fy} , V_{rx} , V_{ry} (which are the horizontal and vertical vector components of the real tyre speed) and contrary to these lateral speeds, F_{fy} (front) and F_{ry} (rear) appear.

In this case, a right-hand curve is being traced, therefore, the car is going "outwards" due to the centrifugal force, but the tyre makes a force "inwards" to try to maintain the trajectory of the curve.

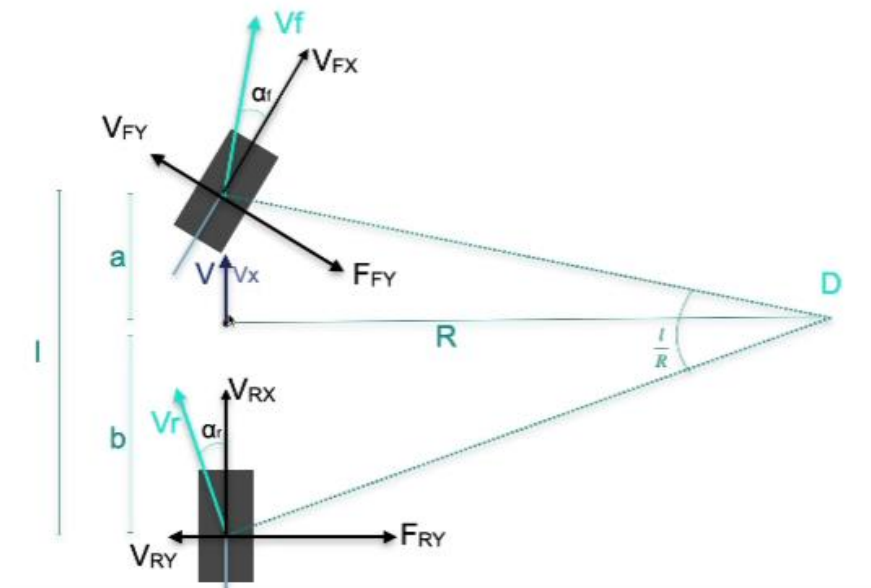


Figure 4.8. Forces acting on the tires.[10]

Because of the equilibrium of moments, there is the following rule:

$$F_{ry} * b = F_{fy} * a$$

What we will now try to explain is the influence of the centre of gravity. As the centre of gravity, a must be smaller than b , which means that F_{fy} must be larger than F_{ry} , so, the force in the front tyre is higher than the force in the rear tyre.

And what happens with a backward centre of mass (Figure 4.9)? Exactly the opposite, now $b < a$, so the force on the rear tyre is greater than the force on the front tyre.

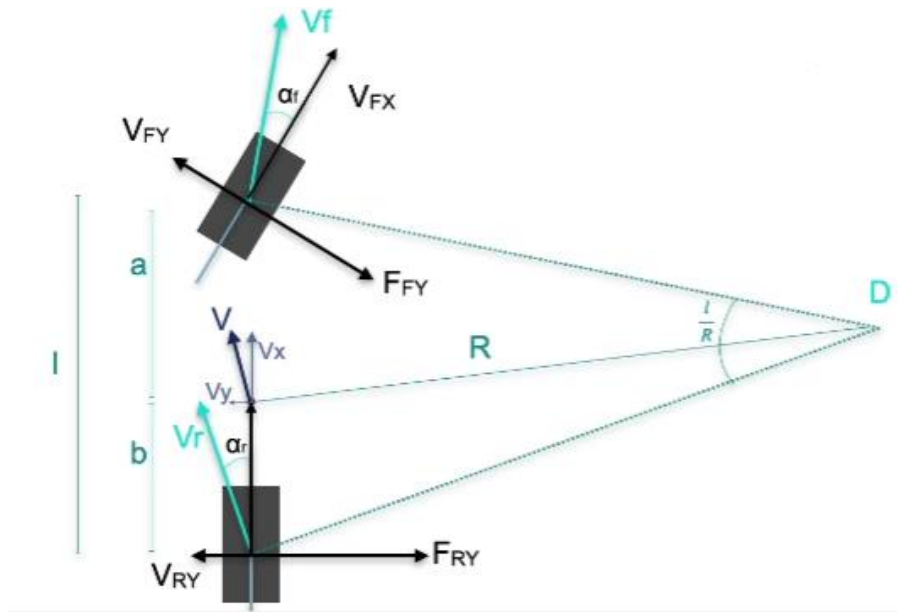


Figure 4.9. Backwarded CG car.[10]

$$F_{RY} * b = F_{FY} * a$$

$$b < a \rightarrow F_{FY} < F_{RY}$$

4.3. High speed corners

Based on figure 4.9 and what was explained in the previous chapter about the backward centre of mass, we can better explain what happens in figure 2.8, which is based on real data from a racing tyre.

For this purpose we will use figure 4.10.

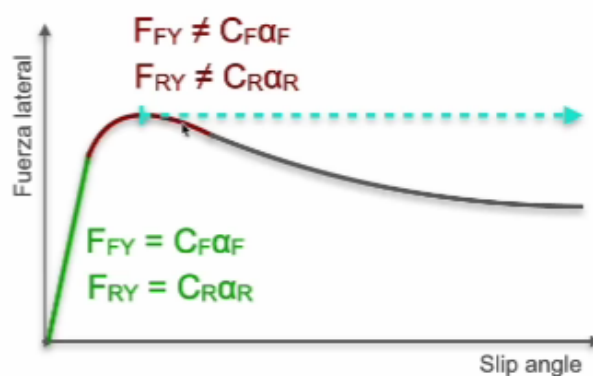


Figure 4.10. Lateral force v slip angle.[10]

As shown by the green line, for each tyre, there is a section where as the slip angle increases, the lateral force of the respective tyre increases. In this green

zone, the lateral force increases linearly, i.e. if the slip angle doubles, the lateral force doubles, multiplying the coefficient (which is given depending on the construction characteristics of the tyre) by the degrees or radians of slip angle.

However, when reaching the red zone, this linear relationship is no longer fulfilled, i.e. there is no longer proportionality between the slip angle and the lateral force.

This has some interesting conclusions. As described above with the following equation:

$$b < a \rightarrow F_{FY} < F_{RY}$$

Therefore:

$$\alpha_r > \alpha_f$$

It follows that it is the rear axle that is at risk.

And what does this mean? It means that there is a very important point just above the red curve, where the blue dashed line starts, because from this point, if the slip angle increases, the lateral force decreases, which contributes to break the moment of balance of forces, which indicates that the grip has been lost in the train that is at risk, in this case in the rear.

And what happens if the centre of mass is advanced as in figure 4.8? We had the following equation:

$$F_{RY} * b = F_{FY} * a$$

and also considering figure 4.10:

$$a < b \rightarrow F_{RY} < F_{FY} \rightarrow \alpha_r < \alpha_f$$

So, now, front axle is at risk. Therefore, the car will tend to slide outwards.

4.4. Oversteer and understeer

When we have a neutral car, i.e. one that neither understeers nor oversteers, the diagram and equations shown in figure 4.11 are useful, where the distance between a and b are equal.

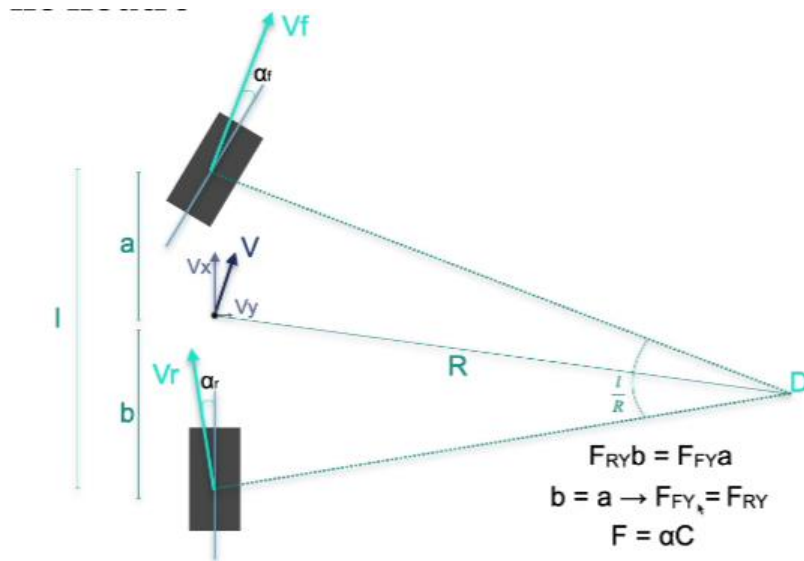


Figure 4.11. Neutral car.[10]

First equation represents that the force exerted by the tyres “inward” (perpendicular to V_f and V_r) are equal for both front and rear tyres as $a=b$.

In addition, third equation remind that the lateral force is the product of the coefficient and the slip angle.

Looking at figures 4.4 and 4.5, from which the following equation was derived:

$$\alpha_r + \vartheta = \alpha_f + \frac{l}{R}$$

When you want to clear the steering wheel rotation angle that the pilot enters in the steering wheel you get:

$$\vartheta = \alpha_f + \frac{l}{R} - \alpha_r$$

And if $\alpha_r = \alpha_f \rightarrow \vartheta = \frac{l}{R}$

And this is what it means to have a neutral car. It can have some implications, for example, if you want to take a very tight corner, R will be very small, and therefore the turning angle you have to give to the steering wheel must be very large.

Let us now look at the example of an understeering car (Figure 4.12).

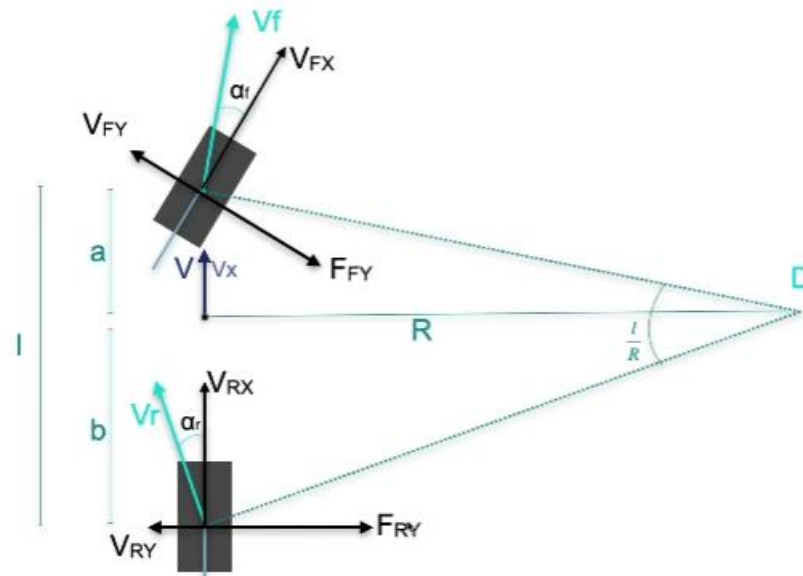


Figure 4.12. Understeering car.[10]

This is the case of the forward centre of gravity, where a is less than b , and we have the following equations:

$$F_{ry} * b = F_{fy} * a$$

$$b > a \rightarrow F_{fy} > F_{ry}$$

$$F = \alpha C$$

As seen in the previous chapter, for a forward CG vehicle, the force exerted by the front tyres are higher than the force exerted by the rear tyres. As for the neutral car, reminding figures 4.4 and 4.5:

$$\delta = \alpha_f - \alpha_r + \frac{l}{R}$$

$$\alpha_f > \alpha_r \rightarrow \delta > \frac{l}{R}$$

So, in conclusion, the driver has to give a greater steering angle. The understeering car tends to run “outwards”.

Finally, let's look at the example of an oversteering car. For this we take as an example figure 4.9 (vehicle with a retarded centre of gravity), whereby a is greater than b and the next equations are satisfied

$$F_{RY} * b = F_{FY} * a$$

$$b < a \rightarrow F_{FY} < F_{RY}$$

$$F = \alpha C$$

Again, using figures 4.4 and 4.5, we arrive at the following equations:

$$\partial = \alpha_f - \alpha_r + \frac{l}{R}$$

$$\alpha_f < \alpha_r \rightarrow \partial < \frac{l}{R}$$

So, in conclusion, and the opposite of an understeering car, the driver has to give less steering angle. The oversteering car tends to run “inwards”.

4.5. Turning equations and graphics

We will start by establishing a lateral basis, i.e. a basis according to the lateral dynamics.

$$\partial = \alpha_f - \alpha_r + \frac{l}{R}$$

This equation can be simplified in:

$$\partial = K a_y + \frac{l}{R}$$

We already know that α_r and α_f depends on the speed at the entry of the turn, the higher the speed (higher a_y), the greater the slip angles, because it depends on the lateral acceleration (a_y). K will be an important factor when determining how much oversteer or understeer is a vehicle.

We need to define as well that the centrifugal force is related with the forces exerted by the front and rear tyres (figure 4.13):

$$\frac{mV^2}{R} = F_{YF} + F_{YR}$$

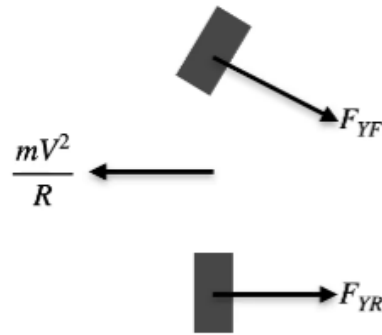


Figure 4.13. Centrifugal force.[10]

$$F_{YR} * b = F_{YF} * a$$

This equation has to be satisfied as well, where a is the distance from the front tyre to the center of mass and b from the rear tyre to the center of mass.

These equations can be combined to obtain the following equations:

$$\frac{mV^2}{R} = F_{YR} + F_{YR} \frac{b}{a}$$

$$\frac{mV^2}{R} = F_{YR} \left(1 + \frac{b}{a}\right)$$

$$\frac{mV^2}{R} = F_{YR} \left(\frac{a+b}{a}\right)$$

$$\frac{mV^2}{R} = F_{YR} \frac{l}{a}$$

$$F_{YR} = \frac{mV^2}{R} \frac{a}{l} \quad F_{YF} = \frac{mV^2}{R} \frac{b}{l}$$

A vertical base now needs to be established according to the scheme shown in figure 4.14.

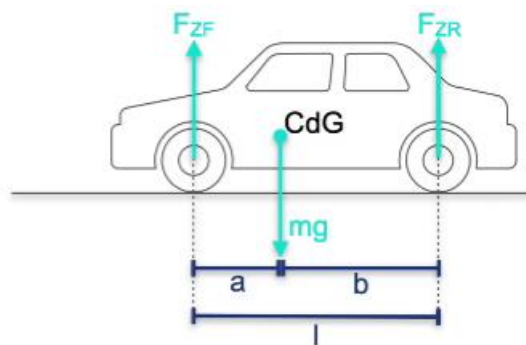


Figure 4.14. Vehicle scheme.[10]

In order to maintain the balance of moments it must be complied with:

$$F_{ZR}l = mga$$

If $F_{ZR}l$ were higher than mga , the car would be lifted by the rear end.

The following equations can be derived from the above equation:

$$F_{ZR} = mg \frac{a}{l}$$

$$\frac{F_{ZR}}{g} = \frac{ma}{l} \quad \text{or} \quad \frac{F_{ZF}}{g} = \frac{mb}{l}$$

Knowing that:

$$F_{YR} = \frac{mV^2}{R} \frac{a}{l}$$

Then:

$$F_{YR} = \frac{V^2}{R} \frac{ma}{l} \quad ;$$

$$F_{YR} = \frac{V^2}{R} \frac{F_{ZR}}{g}$$

And:

$$F_{YR} = C_R \alpha_R$$

So:

$$C_R \alpha_R = \frac{V^2}{R} \frac{F_{ZR}}{g} \quad ;$$

$$\alpha_R = \frac{V^2}{RC_R} \frac{F_{ZR}}{g} \quad \text{or} \quad \alpha_F = \frac{V^2}{RC_F} \frac{F_{ZF}}{g}$$

Now, with all the equations obtained so far, we will try to account for the K factor mentioned at the beginning of this subsection.

$$\alpha_F = \frac{V^2}{RC_F} \frac{F_{ZF}}{g}$$

$$\alpha_R = \frac{V^2}{RC_R} \frac{F_{ZR}}{g}$$

$$\partial = \alpha_f - \alpha_r + \frac{l}{R}$$

Substituting:

$$\partial = \frac{l}{R} + \frac{V^2}{RC_F} \frac{F_{ZF}}{g} - \frac{V^2}{RC_R} \frac{F_{ZR}}{g}$$

$$\partial = \frac{l}{R} + \frac{F_{ZF}}{C_F} \frac{V^2}{Rg} - \frac{F_{ZR}}{C_R} \frac{V^2}{Rg}$$

$$\delta = \frac{l}{R} + \left(\frac{F_{ZF}}{C_F} - \frac{F_{ZR}}{C_R} \right) \frac{V^2}{Rg}$$

$$\delta = \frac{l}{R} + \left(\frac{F_{ZF}}{C_F} - \frac{F_{ZR}}{C_R} \right) a_y$$

And this last equation is similar to the one defined at the beginning of this subsection ($\delta = K a_y + \frac{l}{R}$), so we can conclude that:

$$K = \frac{F_{ZF}}{C_F} - \frac{F_{ZR}}{C_R}$$

K is the factor relating the vertical force of the front tyre by the coefficient corresponding to that tyre minus the vertical force of the rear tyre by the coefficient of the rear tyre. Therefore understeer and oversteer depend on the position of the centre of mass and the weight of the vehicle.

Attention should be paid to the units for the equations defined above.

Angles (δ, α) should be in radians.

Masses should be in kilograms.

Length in meters.

Velocities in m/s.

Forces in Newtons.

K in rad/g.

Figure 4.15 shows a graph relating flywheel rotation (ordinate axis) and speed (abscissa axis). It can be seen that for a positive K, the flywheel rotation must be greater, and that for a negative K, the flywheel rotation is reduced, and may even reach a flywheel rotation equal to zero, which would indicate the critical speed, as this turn could not be taken.

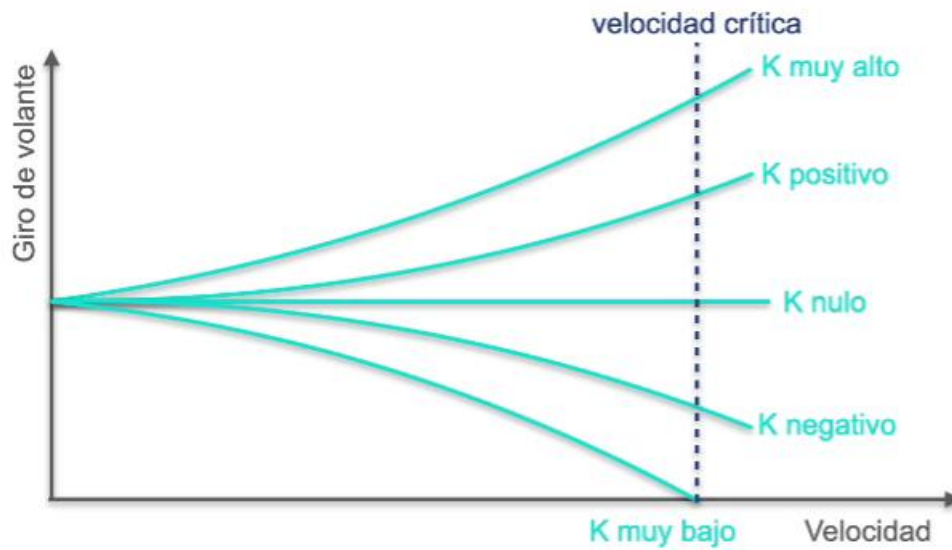


Figure 4.15. Flywheel rotation v speed.[10]

It is noted that this equation can be positive or negative. If the tyre stiffnesses were the same at the front and rear axles (C_F, C_R), if F_{ZR} were greater than F_{ZF} , we would have the case of a CG towards the rear, and K would be negative. In the opposite case, if F_{ZF} were greater than F_{ZR} , the centre of gravity would be towards the front and K would be positive.

Now analysing the value of K , if the car were to move with a speed such that $\frac{V^2}{R}$ equals one g, the value of K would give the Ackermann steering correction that applies to one g. Therefore, a positive value of K indicates an understeering car, a negative value of K indicates an oversteering car and the value gives the number of degrees per unit a_Y of steering deviation from the neutral or Ackermann steering geometry that is applied.

4.5.1. Other contributions to the understeer gradient (K)

We have already seen that the positioning of the centre of gravity affects vehicle behaviour, but there are even more factors that contribute to vehicle performance.

Consider figure 4.16, with the lateral force at the front and rear in momentum balance with each other, it is clear that this momentum balance is the cause of the understeer and oversteer effects discussed above. However, if there is something else that affects the magnitude of the lateral force generated at the front and rear,

this will also affect the momentum balance and therefore has the ability to affect whether the car is understeering or oversteering.

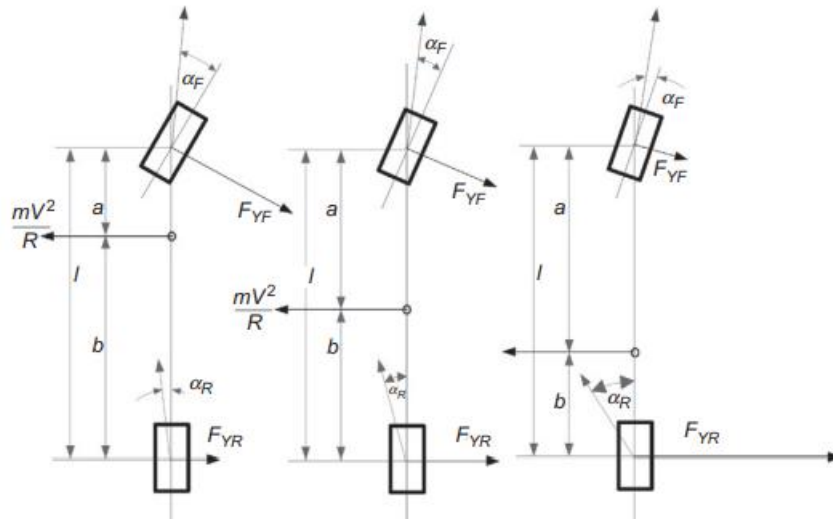


Figure 4.16. Three regimes of steering response.[11]

Fitting certain types of suspension affects the degree of understeer of the vehicle. However, also elements that have nothing to do with the suspension have a good influence on understeer. For example, a vehicle in which the centre of pressure is located at the rear will experience oversteer when a gust of wind comes from the side and will understeer because the driver will need to turn the steering wheel less due to the fact that the gust has already provided part of the steering input.

We conclude that the amount of understeer does not depend solely on the position of the centre of gravity and we need a more general way of dealing with the amount of understeer. We do this by making use of an alternative form of the cornering equation:

$$\delta = \frac{l}{R} + K a_y$$

In this version, we have the steered angle, wheelbase and corner radius as before, but now, the quantity K is introduced. This is called the understeer gradient and is a summation of all the different effects that contribute to the amount of understeer. It is called the understeer gradient because as we can see it determines whether the graph of steered angle against forwards speed has a slope that increases or decreases as acceleration increases and so it controls the gradient of this line. [12]

Knowing that:

$$K = \frac{F_{ZF}}{C_F} - \frac{F_{ZR}}{C_R}$$

By equilibrium of moments we obtain:

$$K = \frac{mg}{l} \left(\frac{a}{C_R} - \frac{b}{C_F} \right)$$

But if we were wanting to consider other sources of understeer, then we should need to determine other terms for K to be added to that above

$$K = \left(\frac{F_{ZF}}{C_F} - \frac{F_{ZR}}{C_R} \right) + ?_{ROLL STEER} + ?_{CAMBER GAIN}$$

When a vehicle rolls as a result of lateral acceleration, it is very likely that due to the design of the suspension, toe changes will occur at one end and then the other. For example when a car has a McPherson strut at the front and a wishbone at the rear, the toe change is different at each end, so changes in the front and rear slip angle will occur. A relationship can be noted between roll angle and the difference in lateral forces and also a relationship between understeer gradient and roll angle.

As in the previous paragraph, due to the roll, a difference between the camber at the front and rear axles will cause a change in the lateral forces at each end. So, a relationship between camber gain and understeer gradient can be established.

The forward displacement of the centre of pressure due to the presence of the front wing on racing cars and the increased aerodynamic load in that area with increasing speed, as well as the tolerances in the mounting of the steering rack and suspension can also be determinant for changes in the forces acting on the tyres. Their calculation is beyond the scope of this project and will therefore not be discussed.

4.5.2. Differentials

A differential is a mechanism that allows the tractive force to be divided between two output shafts starting from an input shaft, allowing these two output shafts to rotate at different speeds. [13] The reason for this is that when cornering, the outside wheels do not travel the same distance as the inside wheels. The inside wheels travel a shorter distance and, if there were no differential, they would be

dragged by the outside wheels which would force them to turn faster than necessary. [14]

With an open differential, during a corner, if one of the two wheels has more resistance to turn (it would be the outside wheel, as it is the most loaded due to weight transfer), the satellite yields to this greater resistance, rotating on itself and sending a greater number of turns to the opposite wheel via the planetary gears. This need not be a problem for curves where there is not a large weight transfer. However, if one drive wheel is in the air (as can happen with the inside wheel during a corner) or if it is on a slippery surface and the other is not, the car will not move forward because the differential will send close to zero torque to both wheels (the torque that the wheel in the air can handle). [15]

To solve this, one could think that the best solution could be the absence of the differential and keeping a solid axle to transmit the same power to both wheels on each side. This could be useful in low grip or full weight transfer situations. However, at modest speeds, where there needs to be a difference in rotational speed between the wheels of the drive axle, allowing the outside wheel to complete a larger distance, it does not solve the problem. In addition, a solid axle is a problem from a handling point of view as the inside wheel is forced to turn faster than the curve radius itself demands which will induce a longitudinal slip, producing a greater longitudinal forward force than is generated on the outside wheel. The resulting torque will urge the car to turn out of the corner and continue straight ahead of the corner and continue straight ahead. Thus, a solid rear axle is a strongly understeering effect.

The differentials used in Formula 1 and also in the World Rally Championship (WRC) are electrohydraulic, which are very expensive. For this reason, this type of differentials are not fitted on road cars and are only used in the above categories. Information about these differentials are sparse, and Wright gives a broad overview of them [16].

To solve the problem generated by the open differentials and the rigid axle, Limited Slip Differentials (LSD) are used. These are divided into two subcategories, torque-sensitive and speed-sensitive. Both transfer torque to the

wheel that has more traction from the wheel that has less traction through the differential, providing a traction improvement over what the open differential would be capable of. In these, what is done is to allow a maximum turning differential, beyond which the differential locks. This maximum permissible turning differential is called the "self-locking differential setting" and is indicated by two figures which refer to the percentage by which the differential locks under acceleration and in lock-up. A 0/0 differential would be a conventional differential that locks neither under acceleration nor under lock-up (if one wheel is turning 100 turns, the one on the other side could be turning 0). A 100/100 differential would be basically welding one wheel to the other, not allowing any turning differences between the two. A 25/40 differential would be one where if the wheel on one side turns 100 turns, the other would turn 25 (and in lock-up, 40).

In racing cars, locking is often used in excess of 40%, however, in road cars these settings would be too abrupt, so they usually do not exceed 25% - unless they are active differentials with electronic control, in which the setting is variable depending on the conditions. [17]

Another solution could be the Torsen differential, which provides proportional torque between the drive axles by interconnecting the drive axles with a helical-type gear arrangement. This arrangement is designed to support a predetermined torque ratio between the drive axles. [18]

It is obvious that the differential situated on the rear axle can be used in a manner to affect the vehicle dynamics. The use of mechanical forms of clutch pack LSD differentials, which have an inherent resistance to turning caused by the preload torque, was deemed disadvantageous with respect to torque transfer provided by differential braking via the ABS control mechanism [19]. However, the use of LSDs for controlling vehicle stability has been explored. One set of researchers has made the case for using a Torsen differential instead of differential braking for affecting vehicle stability and complementing a traction control system [20].

Since we are concerned here with the effect on handling, we will only consider the case of a vehicle with an open differential and a solid axle. These two extremes provide limits between which all differentials will lie. [11]

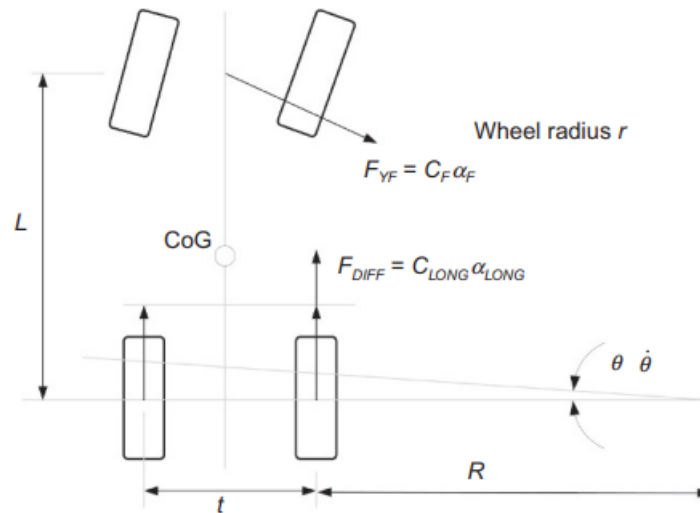


Figure 4.17. Effect of differential on understeer gradient.[11]

In figure 4.17, the vehicle is cornering to the right, so the weight is transferred to the outside wheels. The outside wheel has good grip on the road, and because it is a solid axle, the inside wheel is rotating faster than necessary. Therefore, it has a longitudinal slip and produces a longitudinal force F_{DIFF} , in addition to the force it is producing because of the torque it is bearing.

Knowing that the forwards velocity of the inner wheel is $R\dot{\theta}$ and the forwards velocity of the outer wheel is $(R+t)\dot{\theta}$, can be deduced that the angular velocity of the outer wheel is $\frac{v}{r} = \frac{(R+t)\dot{\theta}}{r}$.

The inner wheel is obliged to rotate with the same angular velocity as the outer wheel, and establishing that the longitudinal slip ratio is given by the formula

$\left(1 - \frac{r\omega}{v}\right) \times 100$, we have:

$$\text{Longitudinal slip ratio} = 1 - \frac{(R+t)\dot{\theta}}{R\dot{\theta}} \times 100 = \frac{t}{R} \times 100$$

By taking moments now it will be possible to calculate how much lateral force needs to be generated at the front axle to balance the yawing torque caused by F_{DIFF} .

$$C_a \alpha_F L = F_{DIFF} \frac{t}{2};$$

$$\alpha_F = \frac{1}{C_a L} C_{LONG} \alpha_{LONG} \frac{t}{2};$$

$$\alpha_f = \frac{C_{LONG}}{C_a L} \frac{t}{R} \frac{t}{2} \times 100;$$

$$= \frac{C_{LONG} t^2}{2 C_a L R} \times 100$$

Therefore, it can be said that a slip angle α_f is needed to compensate for the understeer effect caused by the solid rear axle. Evidently, more would be needed to make the car take the corner. Let's look at the example for a Formula Student on a 20 m skid pan.

$$C_{LONG}: 1100$$

$$C_{lat}: 1200 \frac{N}{degree}$$

$$R: 20m$$

$$T: 1.5m$$

$$L: 1.6m$$

$$\alpha_f = \frac{C_{LONG} t^2}{2 C_a L R} = \frac{1100}{2 \times 1200 \times 1.6} \frac{1.5^2}{20} \times 100 = 3.2 \text{ degrees}$$

The size of this steered angle is the cause for concern. Most racing tyres develop their peak lateral force at around 8–10 degrees of slip. This means that a car without a differential would already be using up 2.6 degrees of slip in countering the effect of the differential and only around 6.4 will be left for cornering. If the car were fitted with a limited-slip differential, this would be much improved. [11]

From this it can be concluded that if this vehicle were equipped with a solid rear axle, about one third of the lateral force generation capacity would be lost, whereas with an open axle nothing would be lost. However, the solid rear axle would still provide torque to the outer wheel even in the event of the inner wheel

lifting. We must find the middle ground between these two extremes to improve performance. This is not a simple task. A cornering analysis based on the derivative analysis to be explained later must be performed which incorporates a loss of cornering force on the front wheels depending on the F_{DIFF} force. A naturally neutral car will potentially be faster because no longitudinal torque is wasted to correct the understeer gradient. For road cars, this is not of great importance, because making a car neutral can be complicated for design reasons and limited-slip differentials are more often used.

4.6. Equations of motion for a cornering racing car

In the next chapter we will develop equations that describe the response of a vehicle to a step-steering input. Figure 4.18 shows the dynamics of a vehicle during a curve using the bicycle model. Initially, the car is in the "0.0 seconds" position. The car is moving in a straight line, without any steering input. The "trail line" showing backwards from the centre of gravity shows where the car has just been.

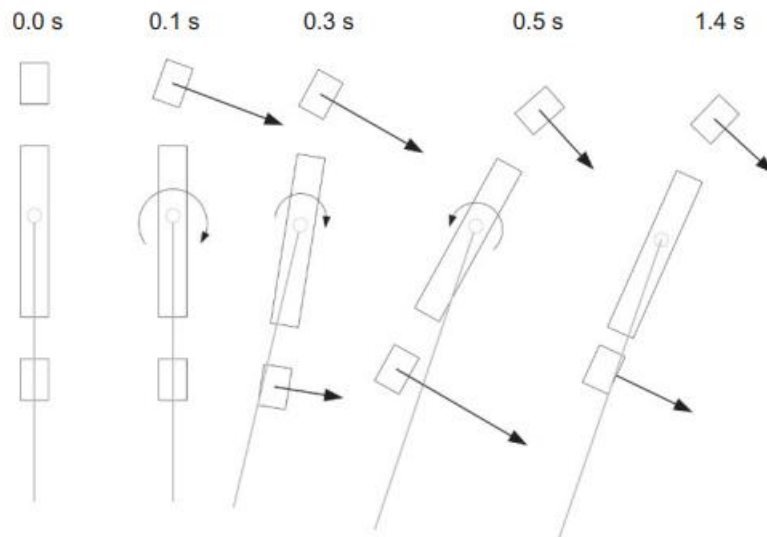


Figure 4.18. Evolution of side slip, yaw velocity and yaw displacement.[11]

At position "0.1 s" the driver applies a rapid steer input to the steering. The front wheel now has a small slip angle, and generates a lateral force. The car has not yawed too much because the force has not been applied for a long time, but it has a small yaw velocity, illustrated by the rotational arrow. Immediately after, at the

"0.3 s" instant, the lateral force on the front tyre has caused the whole vehicle to yaw. A slip angle is generated on the rear wheel. The radius of the curve is still very large, so the lateral force needed to keep the vehicle in the curve is very low, lower than the sum of the lateral forces generated by the front and rear tyres. In the "0.5 s" position the vehicle has yawed more and has passed the equilibrium position. The yaw moment is used to accelerate the car in the other direction as there is reduced force at the front and increased lateral force at the rear. In the "1.4 s" position the vehicle is in equilibrium, the moment produced by the front and rear lateral forces is zero, the yaw velocity is constant and the side slip velocity is constant.

Figure 4.19 shows a graph showing the yaw velocity (solid line) and yaw displacement (dotted line). The time on the bottom axis is the same as in figure 4.18. The yaw velocity is seen to exceed the equilibrium value and then re-establish itself in the same way as a one degree of freedom resonant system would. The yaw displacement is a straight line of positive gradient, which is to be expected for a car in constant circular motion.

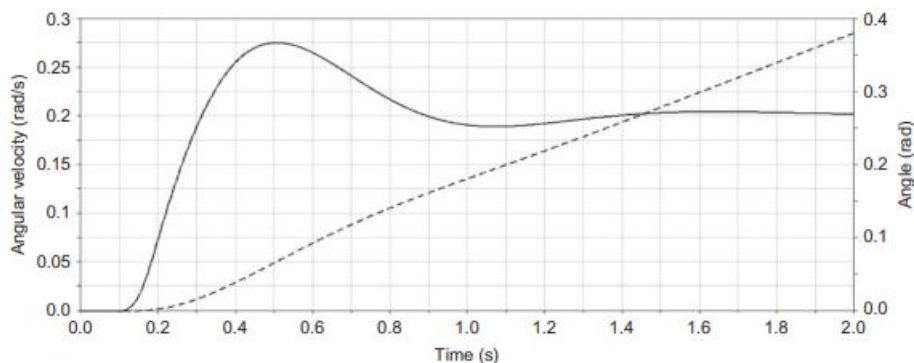


Figure 4.19. Yaw rate and yaw angle response to step steer.[11]

As mentioned above, a neutral steering car is best if you are looking to maximise performance. However, the considerations made so far are very steady state when the dynamics presented are clearly transient, i.e. only present for a short period of time. Being able to model such transient dynamics would be very beneficial. For example, it can be seen that there is a minimum time in which any vehicle can complete this transient process, and it cannot be improved further. Let's take the case of a race engineer talking to his driver, who complains that the car is too slow

to respond to corner entry. If the engineer were to make changes to the car's set-up, he might be wasting his time. He should do the derivative analysis that will now be introduced to determine how close he is to the theoretical best time the car can do. If the driver meets that time, then modifications should be made to the car.

In figure 4.20 the scheme is similar to the one shown in previous sections but with more details and parameters to consider. The car travels around a curve of radius R , at a speed V , which is tangential to the line from the centre of gravity to the centre of the curve. This velocity is divided into two components, the lateral velocity V_y and the forward velocity V_x . β represents the slip angle of the vehicle. In addition, the car has a yaw velocity. The yaw rate is r , and is located above the centre of gravity. At the front, the slip angle α_F applies and results in a force on the front tyre. α_R applies in a similar way at the rear. N is the total yawing moment, which results from adding each lateral force by its perpendicular distance from the centre of gravity. The total lateral force will be called Y .

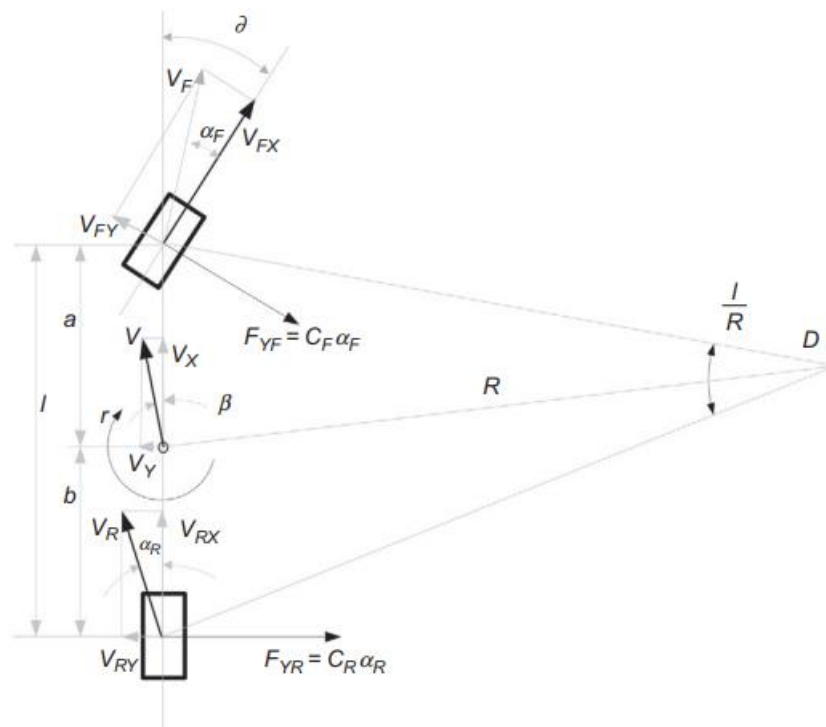


Figure 4.20. Free body diagram for a cornering vehicle. Note that in the picture ∂ is the same as δ in the following equations.[11]

4.6.1. Lateral Force Equilibrium

The lateral acceleration of the car in a curve is described by:

$$a_Y = \frac{V^2}{R}$$

The angular velocity about the centre of the curve is equal to the yaw velocity.

$$V = \omega R; V = rR; \frac{V}{R} = r;$$

Thus, the lateral acceleration that results from rotation about the corner centre is:

$$a_Y = \frac{V^2}{R} = \frac{V}{R} V; a_Y = rV;$$

The car has a horizontal side slip velocity, V_Y , which changes constantly during the course of the curve, so we have a horizontal acceleration component:

$$a_Y = \dot{V}_Y;$$

The lateral acceleration is the sum of these two components and

$$a_Y = Vr + \dot{V}_Y;$$

Because the angles are small and the velocity V is constant,

$$V_Y = V \sin \beta; V_Y \approx V \beta; \dot{V}_Y = \frac{d}{dt}(V \beta) = V \dot{\beta};$$

Lateral acceleration is:

$$a_Y = V(r + \dot{\beta});$$

Now we look at the rear of the vehicle to analyse α_R . The tyre slides to the right because the whole vehicle slides with speed V_Y . Also, it slides to the left because the whole vehicle is yawing about the centre of gravity. The magnitude of this component is $r \times b$. The lateral velocity is therefore

$$V_{YR} = V_Y - rb$$

By definition, the slip angle at the rear is given by

$$\alpha_R = \tan^{-1}\left(\frac{V_{YR}}{V_X}\right) \approx \frac{V_{YR}}{V_X};$$

So,

$$\alpha_R = \frac{V_Y - rb}{V_X}$$

Approximately,

$$\beta = \tan^{-1}(V_Y/V_X) \approx V_Y/V_X$$

the speed in direction x is approximately the same as that of the vehicle, $V_X \approx V$, then

$$\alpha_R = \beta - \frac{rb}{V}$$

A similar analysis is made at the front, although here the yaw velocity moves from the front to the right, not to the left and the steered angle δ . The velocity on the front V_{YF} tyre is:

$$V_{YF} = V_Y + ra$$

Since the projection of V_{YF} on the y-axis is approximately V_{YF} and taking into account that α_F is negative, the angle between the front wheel velocity, V_F and the x-axis, $(\delta - \alpha_F)$, can be approximated to:

$$\begin{aligned} \delta + \alpha_F &= \tan^{-1} \left(\frac{V_{YF}}{V_X} \right) \\ &\approx \frac{V_{YF}}{V_X} \\ \rightarrow \alpha_F &= \frac{V_{YF}}{V_X} - \delta \end{aligned}$$

Substituting for V_{YF}

$$\alpha_F = \frac{V_Y + ra}{V_X} - \delta$$

Which is simplified to

$$\alpha_F = \beta + \frac{ra}{V} - \delta$$

We can calculate the lateral forces front and rear, because we know the slip angle of the front and rear tyre, and α_F is in the sense shown in figure 4.20.

$$F_{YF} = C_F \left(\beta + \frac{ar}{V} - \delta \right)$$

$$F_{YR} = C_R \left(\beta - \frac{rb}{V} \right)$$

Lateral force is

$$F_{TOT} = C_F \left(\beta + \frac{ar}{V} - \delta \right) + C_R \left(\beta - \frac{rb}{V} \right)$$

$$\rightarrow F_{TOT} = (C_F + C_R)\beta + C_F \frac{ar}{V} - C_R \frac{rb}{V} - C_F \delta$$

Since the total lateral acceleration is given by

$$a_Y = V(r + \dot{\beta})$$

We can write

$$mV(r + \dot{\beta}) = (C_F + C_R)\beta + \frac{r}{V}(C_F a - C_R b) - C_F \delta$$

4.6.2. Moment equilibrium

The total momentum that makes the car to yaw is called yawing moment. In a straight line or in a steady-state corner, this moment is zero because the car is not accelerating, but on corner entry and exit, the yawing moment is large.[11]

$$N = N_F + N_R$$

$$\rightarrow N = F_{YF}a - F_{YR}b;$$

$$N = C_F a \beta + C_F \frac{a^2 r}{V} - C_F a \delta - C_R b \beta + C_R \frac{r b^2}{V}$$

The total yawing moment accelerates the car in yaw according to $T = I\ddot{\theta}$. Where r is the yaw rate, and \dot{r} is the yaw acceleration.

$$I\dot{r} = (C_F a - C_R b)\beta + \frac{r}{V}(C_F a^2 + C_R b^2) - C_F a \delta$$

I is the polar moment of inertia about the vertical axis of the car through the centre of gravity.

Figure 4.21 shows all the processes carried out so far. Imagine the moment immediately after applying a significant step steer to the front wheel (Figure 4.18, position "0.1 s"). The car will clearly yaw because torque has been applied and it will also accelerate to the right because the centripetal force has not yet acted, the car is still in a straight line and R is infinite.

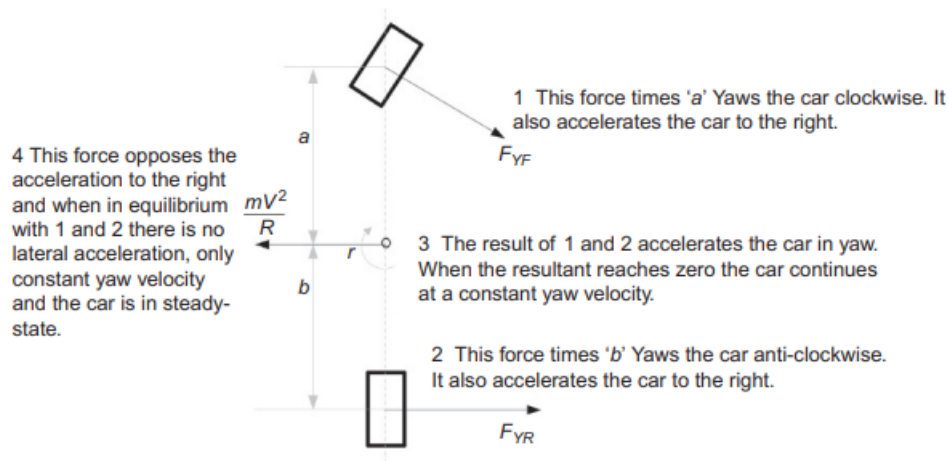


Figure 4.21. Summary of the derivation of the equations of motion.[11]

The lateral force equilibrium and moment equilibrium equations for a vehicle during a curve shown again below are of vital importance. They relate the response of a car to a steering input and when solved will yield a time-based response. A solution can be obtained which relates the radius of the curve to the steering input and will reveal its dependence on speed.

$$mV(r + \dot{\beta}) = (C_F + C_R)\beta + \frac{r}{V}(C_F a - C_R b) - C_F \delta$$

$$I\dot{r} = (C_F a - C_R b)\beta + \frac{r}{V}(C_F a^2 + C_R b^2) - C_F a \delta$$

These equations contain the yaw rate r , the steering input δ and the vehicle side slip angle β . As both equations have β we can obtain a single equation that contains only δ and r . This will allow us for example to determine how long it will take the car to adopt a new heading in response to a given steering input. One might think, and one would be right, that an infinitely short response time would be better, as it would allow for much faster curve negotiation. However, this is impossible to obtain as there will always be an inertia value I . Moreover, it would be absurd to have such short response times that the driver, due to human reaction limits, would not be able to take advantage of them.

We can see that the two fundamental equations shown above are functions of 3 variables, and we can reduce the equations to:

$$mV(r + \dot{\beta}) = f(\beta, r, \delta)$$

$$I\dot{r} = f(\beta, r, \delta)$$

In order to solve these equations we must take the partial derivatives of each independent variable, add them all together and equal them to the change in total lateral force and yawing moment that would result from a change in β , r or δ , at a particular point.

$$\Delta(mV(r + \dot{\beta})) = \left(\frac{\partial Y}{\partial \beta}\right)\beta + \left(\frac{\partial Y}{\partial r}\right)r + \left(\frac{\partial Y}{\partial \delta}\right)\delta$$

$$\Delta I\dot{r} = \left(\frac{\partial N}{\partial \beta}\right)\beta + \left(\frac{\partial N}{\partial r}\right)r + \left(\frac{\partial N}{\partial \delta}\right)\delta$$

The Δ is often omitted in many textbooks, and the resulting expression would be this:

$$mV(r + \dot{\beta}) = Y_{\beta}\beta + Y_R r + Y_{\delta}\delta$$

$$I\dot{r} = N_{\beta}\beta + N_R r + N_{\delta}\delta$$

Direct comparison with the equations of equilibrium shown below gives the equations for each of the derivative terms

$$mV(r + \dot{\beta}) = (C_F + C_R)\beta + \frac{r}{V}(C_F a - C_R b) - C_F \delta$$

$$I\dot{r} = (C_F a - C_R b)\beta + \frac{r}{V}(C_F a^2 + C_R b^2) - C_F a \delta$$

Derivative terms are:

$$Y_{\beta} = C_F + C_R$$

$$Y_R = \frac{1}{V}(C_F a - C_R b)$$

$$Y_{\delta} = -C_F$$

$$N_{\beta} = (C_F a - C_R b)$$

$$N_R = \frac{1}{V}(C_F a^2 + C_R b^2)$$

$$N_{\delta} = -C_F a$$

Each of these derivative terms has a physical meaning in the dynamics of the car. They are described below:

$$Y_{\beta} = C_F + C_R$$

This is the side(lateral) force slope. Similar to the side force slope vs. slip angle for a tyre, but in this case it is the side force slope for the whole vehicle. It is a measure of the rate that lateral velocity is developed as the car is sideslipped. Since the slope Y_β is always negative in the linear range, i.e., a negative lateral force for a positive sideslip velocity, it corresponds to the rate in a linear damper and is called the damping-in-sideslip. [3]

$$Y_R = \frac{1}{V} (C_F a - C_R b)$$

Determine how much lateral acceleration results from each unit of yaw velocity. If one wheel is much further away from the centre of gravity than the other and one moves to the left and one to the right, due to a yaw velocity on the car, the total lateral force will not be in equilibrium and a lateral acceleration will appear.

$$Y_\delta = -C_F$$

Relates the lateral acceleration resulting from each unit of steering input angle applied by the driver. The slip angle of the total vehicle evolves once a steering angle is demanded at the front wheels. When the vehicle is at its side slip angle β , the value of the lateral acceleration is determined by Y_β . However, the parameter that sets how large this force grows to is Y_δ . Clearly it depends on the cornering stiffness at the front wheels and the angle they are steered. The distance between the centre of gravity and the front wheels affects how large the yaw moment is, which in turn affects how quickly the car reacts to steering input.

$$N_\beta = (C_F a - C_R b)$$

Relate how much yaw acceleration results from each degree of side slip of the entire vehicle. This derivative is very important in determining the stability of the vehicle. If N_β is negative, then a small side slip produces a yaw moment that turns the car more at the front than at the rear. This also causes the slip angle to be greater at the front than at the rear. N_β is therefore the oversteer/understeer derivative; if it is zero, there is no yaw acceleration and the car is neutral.

$$N_R = \frac{1}{V} (C_F a^2 + C_R b^2)$$

Relates yaw acceleration to yaw rate. The yaw acceleration resulting from a yaw velocity value is always opposite to the yaw velocity direction. If we try to yaw a car, N_R acts to intercept that yaw input, so it is a source of yaw damping. If an input pulse of yaw acceleration is provided by some external means, the yaw damping will serve to dissipate it and make the yaw displacement tend asymptotically to a limit value. [11] As it depends on the square of the distances of the axes to the centre of gravity, it will be maximum when a and b are equal. As with Y_R , moving the vehicle around a circle reduces the slip angles, so N_R is inversely proportional to forwards velocity.

$$N_\delta = -C_F a$$

It relates yaw acceleration to steering input angles. The yaw moment at the front depends on the angle at which this axle is steered, the cornering stiffness of the tyre and the distance to the centre of gravity from the front axle. An increase in any of these three variables will increase the yaw moment.

4.7. Transient response to control input

4.7.1. Qualitative response to steering input

We have already seen the transient behaviour that a car must make before it reaches its steady state equilibrium. A car that can reach its steady-state position more quickly will allow it to be more agile, providing a quicker response to the driver. A car without oversteer on corner entry is much more effective at pushing the car to its handling limits, as a car with oversteer requires an experienced driver to perform more operations to get the car into the corner. With the derivative analysis performed above, a simple differential equation can be obtained from the two fundamental equations of motion. We eliminate β and a second order differential equation is produced, identical to the one used to describe the motion of a system of one degree of freedom mass, spring and damper. Once this is done, the response of the system in yaw velocity can be determined from the steering inputs, and how fast the car will reach the required yaw velocity.

4.7.2. Derivatives equations

From

$$mV(r + \dot{\beta}) = Y_{\beta}\beta + Y_R r + Y_{\delta}\delta$$

$$I\dot{r} = N_{\beta}\beta + N_R r + N_{\delta}\delta$$

By rearranging the second equation we can find β :

$$\beta = \frac{1}{N_{\beta}}\{I\dot{r} - N_R r - N_{\delta}\delta\}$$

$$\dot{\beta} = \frac{1}{N_{\beta}}\{I\ddot{r} - N_R \dot{r} - N_{\delta}\dot{\delta}\}$$

Substituting into the first equation

$$mV\left(r + \left(\frac{1}{N_{\beta}}\{I\ddot{r} - N_R \dot{r} - N_{\delta}\dot{\delta}\}\right)\right) = Y_{\beta}\left(\frac{1}{N_{\beta}}\{I\dot{r} - N_R r - N_{\delta}\delta\}\right) + Y_R r + Y_{\delta}\delta$$

$$mVr + \frac{mVI}{N_{\beta}}\ddot{r} - \frac{mVN_R}{N_{\beta}}\dot{r} - \frac{mVN_{\delta}}{N_{\beta}}\dot{\delta} = \frac{IY_{\beta}}{N_{\beta}}\dot{r} - \frac{Y_{\beta}N_R}{N_{\beta}}r - \frac{Y_{\beta}N_{\delta}}{N_{\beta}}\delta + Y_R r + Y_{\delta}\delta$$

So,

$$\frac{mVI}{N_{\beta}}\ddot{r} - \left(\frac{mVN_R}{N_{\beta}} + \frac{IY_{\beta}}{N_{\beta}}\right)\dot{r} + \left(mV + \frac{Y_{\beta}N_R}{N_{\beta}} - Y_R\right)r = \frac{mVN_{\delta}}{N_{\beta}}\dot{\delta} + \left(Y_{\delta} - \frac{Y_{\beta}N_{\delta}}{N_{\beta}}\right)\delta$$

Thus,

$$I\ddot{r} - \left(N_R + \frac{IY_{\beta}}{mV}\right)\dot{r} + \left(N_{\beta} + \frac{Y_{\beta}N_R}{mV} - \frac{N_{\beta}Y_R}{mV}\right)r = N_{\delta}\dot{\delta} + \left(\frac{Y_{\delta}N_{\beta}}{mV} - \frac{Y_{\beta}N_{\delta}}{mV}\right)\delta$$

This equation is of the form

$$I\ddot{r} - c\dot{r} + kr = C_1\dot{\delta} + C_2\delta \quad (4.1)$$

Where

$$I = I$$

$$c = -\left(N_R + \frac{IY_{\beta}}{mV}\right)$$

$$k = \left(N_{\beta} + \frac{Y_{\beta}N_R}{mV} - \frac{N_{\beta}Y_R}{mV}\right)$$

$$C_1 = N_{\delta}$$

And

$$C_2 = \left(\frac{Y_{\delta}N_{\beta}}{mV} - \frac{Y_{\beta}N_{\delta}}{mV}\right)$$

Equation (4.1) is the same differential equation that determines the response time of a suspended free mass anchored to the ground by a spring and relates the yaw rate r and its differentials to the steering input displacement and its rate of change with time.

4.7.3. Transient response to a step-steer input

A good response to a step steer will make the car feel good handling. It is taken into account that the steering input rate $\dot{\delta}$ is zero to obtain a step steer. Equation 4.1 is reduced to obtain:

$$I\ddot{r} - c\dot{r} + kr = C_2\delta$$

δ is the step-steering input which is constant over time. The solution is composed of the sum of two parts. The first, the particular integral, is obtained by solving the equation and the second, the complementary function, is obtained by setting the right-hand side to zero. The particular integral, which corresponds to the solution of the value of r , called r_∞ , in steady state, can be obtained, taking into account that in steady state $\ddot{r} = \dot{r} = 0$.

r_∞ is the yaw rate that the car will settle down once the transient response has faded away and is obtained by setting $\ddot{r} = \dot{r} = 0$.

$$kr_\infty = C_2\delta$$

$$r_\infty = \frac{C_2\delta}{k}$$

The final yaw rate at which the system settles once the transient has passed can be easily determined. The complementary function is obtained by setting the right hand side to zero and one has:

$$I\ddot{r} - c\dot{r} + kr = 0$$

A solution of the form is hypothesised:

$$r = e^{st}$$

Where s is a constant to be determined. Substituting:

$$\left(s^2 + \frac{c}{I}s + \frac{k}{I}\right)e^{st} = 0$$

that can only be true if the bracketed term is zero for all t meaning that

$$\left(s^2 + \frac{c}{I}s + \frac{k}{I}\right) = 0$$

Thus, using the equation for the determination of the roots of a quadratic

$$\begin{aligned} s_{1,2} &= \frac{-\frac{c}{I} \pm \sqrt{\left(\frac{c}{I}\right)^2 - 4\frac{k}{I}}}{2} \\ &= -\frac{c}{2I} \pm \sqrt{\left(\frac{c}{2I}\right)^2 - \frac{k}{I}} \quad (4.2) \end{aligned}$$

and we can now substitute to obtain a general solution normalised to r_∞ : [11]

$$\frac{r}{r_\infty} = Ae^{S_1 t} + Be^{S_2 t} \quad (4.3)$$

If this equation were not divided by r_∞ then the solution would make the yaw rate a function of time for the vehicle being analysed. If we want to compare a very heavy car with a single-seater for example, it is clear that its response would be very different. The problem comes when we want to compare one car with another. Dividing the response by its final steady-state value, we should have a graph that fits unity in both cases. Therefore, by comparing the responses we can tell whether either the heavy vehicle or the single-seater is doing the best it can. The unnormalised response is the version that should be used to obtain the actual yaw rate versus time values for the vehicle being analysed. The constants A and B depend upon initial conditions. The constants S_1 and S_2 dictate the nature of the solution. If they happen to evaluate to positive numbers, then there will be two values S_1 and S_2 that will be real numbers, and upon substitution into Eq. (4.3), the yaw rate will exponentially decay to its steady-state value. If the square root in Eq.(4.2) evaluates to a negative number, there will again be two values, but this time, they will be complex numbers, and the solution will be periodic. The value of the damping coefficient that marks the border between these behaviours is the one that pertains when the square root evaluates to zero. In this case, there will only be one common value for S_1 and S_2 . The solution will still be exponential, but it will mark the boundary between the periodic and harmonic solutions. This value of damping is called the critical damping value, C_c , and is given by setting the term inside the square root in Eq. (4.2) to zero, and so, [11]

$$\left(\frac{C_c}{2I}\right)^2 = \frac{k}{I}$$

$$C_c = 2I \sqrt{\frac{k}{I}}$$

If we consider an undamped free system,

$$I\ddot{r} + kr = 0$$

Rearranging,

$$\omega_n^2 = \frac{k}{I}$$

$$\frac{1}{\omega_n^2} \ddot{r} + r = 0$$

This equation has a harmonic solution of the form $r = A\sin\omega_n t + B\cos\omega_n t$; so a periodic solution for a system with zero damping will be:

$$\omega_n = \sqrt{\frac{k}{I}}$$

Also:

$$C_c = 2I\omega_n$$

The ratio of system damping to critical damping is the damping ratio ζ

$$\zeta = \frac{c}{C_c}$$

Noting that

$$\frac{c}{2I} = \frac{\zeta C_c}{2I} = \frac{2I\omega_n\zeta}{2I} = \omega_n\zeta$$

Re-expressing terms,

$$s_{1,2} = -\omega_n\zeta \pm \sqrt{(\omega_n\zeta)^2 - \omega_n^2}$$

$$s_{1,2} = \omega_n \left(-\zeta \pm \sqrt{\zeta^2 - 1} \right)$$

$$s_{1,2} = \omega_n \left(-\zeta \pm i\sqrt{1 - \zeta^2} \right)$$

We can now substitute these for s_1 and s_2 into the general solution and produce an equation of motion for the three cases: underdamped ($\zeta < 1$), critically damped ($\zeta = 1$) and overdamped ($\zeta > 1$). [11]

4.7.4. Underdamped, critically damped and overdamped steering

Let's begin analysing the first case, underdamped. We start by considering the unnormalised response, so,

$$r = Ae^{(-\omega_n\zeta + i\sqrt{(1-\zeta^2)}\omega_n)t} + Be^{(-\omega_n\zeta - i\sqrt{(1-\zeta^2)}\omega_n)t}$$

$$r = e^{-\zeta\omega_n t} \left(Ae^{i\sqrt{(1-\zeta^2)}\omega_n t} + Ae^{-i\sqrt{(1-\zeta^2)}\omega_n t} \right)$$

Since

$$e^{i\theta} = \cos\theta + i\sin\theta$$

The term in brackets on the right equals real and imaginary components. They can be simplified to a phase angle whose magnitude and phase angle are determined from the initial conditions.

$$r = Xe^{-\zeta\omega_n t} \sin\left(\sqrt{(1-\zeta^2)}\omega_n t + \Phi\right)$$

This is a sinusoidal oscillatory motion away from the input motion by a phase angle Φ which falls exponentially. The frequency of this motion is called the damped natural frequency, ω_d and is given by the following expression

$$\omega_d = \sqrt{(1-\zeta^2)} * \omega_n$$

The total solution is obtained from the sum of the particular integral and the complementary function. The yaw rate response to a step steer for an underdamped vehicle is:

$$r = Xe^{-\zeta\omega_n t} \sin(\omega_d t + \Phi) + \frac{c_2}{K} \delta_0 \quad (4.4)$$

Where X and Φ are the constants determined from the initial conditions. In this case, the car is responding to a step-steer input; the initial yaw rate will be zero, $t = 0 \rightarrow r = 0$. The initial yaw acceleration can be obtained from the initial yaw moment. At the moment at which the steering input is applied, it will come from the lateral force on the front tyres times the distance a . So,

$$T_0 = C_f \partial a$$

This initial torque, at $t=0$, results in an initial acceleration of

$$-\dot{r}_0 = \frac{C_f \partial a}{I}$$

We may now substitute for these initial conditions and find the constants X and ϕ .
Substituting $t=0$ and $r=0$,

$$0 = X \sin(\phi) + \frac{C_2}{K} \delta_0$$

$$X = \frac{-C_2 \partial}{K \sin \phi}$$

To determine ϕ , we start by differentiating Eq. (4.4) above to give

$$\dot{r} = -\zeta \omega_n X e^{-\zeta \omega_n t} \sin(\omega_d t + \phi) + X \omega_d e^{-\zeta \omega_n t} \cos(\omega_d t + \phi)$$

Substituting at $t=0$,

$$\begin{aligned} -\frac{C_f \partial a}{I} &= -\zeta \omega_n X \sin \phi + X \omega_d \cos \phi \\ -\frac{C_f \partial a}{I} &= \frac{\zeta \omega_n C_2 \partial \sin \phi}{K \sin \phi} + \frac{-C_2 \partial \omega_d \cos \phi}{K \sin \phi} \\ -\frac{C_f \partial a}{I} &= \frac{\zeta \omega_n C_2 \partial}{K} + \frac{-C_2 \partial \omega_d}{K} \\ \frac{C_2 \partial \omega_d}{K \tan \phi} &= \frac{C_f \partial a}{I} + \frac{\zeta \omega_n C_2 \partial}{K} \\ \tan \phi &= \frac{\left\{ \frac{C_2 \partial \omega_d}{K} \right\}}{\left\{ \frac{C_f \partial a}{I} + \frac{\zeta \omega_n C_2 \partial}{K} \right\}} \end{aligned}$$

Multiplying top and bottom by KI and cancelling δ ,

$$\tan \phi = \frac{C_2 \omega_d I}{\{C_f a K + \zeta \omega_n C_2 I\}}$$

Thus, the derivative analysis models the yaw rate response to a step-steer input for the underdamped steering system with the following equation:

$$r = X e^{-\zeta \omega_n t} \sin(\omega_d t + \phi) + \frac{C_2}{K} \delta_0$$

Where

$$\begin{aligned} \tan \phi &= \frac{C_2 \omega_d I}{\{C_f a K + \zeta \omega_n C_2 I\}} \\ X &= \frac{-C_2 \partial}{K \sin \phi} \end{aligned}$$

$$K = \left(N_{\beta} + \frac{Y_{\beta} N_R}{mV} - \frac{N_{\beta} Y_R}{mV} \right)$$

$$\phi = \tan^{-1} \left(\frac{\omega_d}{\zeta \omega_n} \right)$$

$$C_c = 2I\omega_n$$

$$c = - \left(N_R + \frac{IY_{\beta}}{mV} \right)$$

$$\zeta = \frac{c}{C_c}$$

$$\omega_n = \sqrt{\frac{k}{I}}$$

$$\omega_d = \sqrt{(1 - \zeta^2)} \times \omega_n$$

The second case, critically damped is with the damping ratio $\zeta = 1$, has the general solution:

$$s_1 = s_2 = -\omega_n$$

$$r = Ae^{-\omega_n t} + Be^{-\omega_n t}$$

$$r = Ce^{-\omega_n t}$$

$C=A+B$. We use the following solution:

$$r = (A + Bt) e^{-\omega_n t}$$

For the equation shown below, when $\zeta = 1$, is given by the sum of the particular integral and the complimentary function:

$$I\ddot{r} - c\dot{r} + kr = C_2\delta$$

So,

$$r = (A + Bt)e^{-\omega_n t} + \frac{C_2\delta}{k}$$

With $t=0$ and $r = 0$,

$$0 = A + \frac{C_2\delta}{k}$$

$$A = -\frac{C_2\delta}{k}$$

The consideration of $t=0$ also implies

$$\begin{aligned}\dot{r}_0 &= -\frac{C_f \partial a}{I} \\ r &= Ae^{-\omega_n t} + Bte^{-\omega_n t} \\ \dot{r} &= -\omega_n Ae^{-\omega_n t} + Be^{-\omega_n t} - \omega_n Bte^{-\omega_n t} \\ -\frac{C_f \partial a}{I} &= -\omega_n A + B \\ B &= -\frac{C_f \partial a}{I} - \omega_n \frac{C_2 \partial}{k}\end{aligned}$$

Thus,

$$r = \left(-\frac{C_2 \partial}{k} + \left\{ -\frac{C_f \partial a}{I} - \omega_n \frac{C_2 \partial}{k} \right\} t \right) e^{-\omega_n t} + \frac{C_2 \partial}{k}$$

Dividing this by r_∞ this equation can be normalised,

$$\frac{r}{r_\infty} = \left(-1 + \left\{ -\frac{C_f \partial a}{I r_\infty} - \omega_n \right\} t \right) e^{-\omega_n t} + 1$$

This therefore relates the yaw rate response of a vehicle with a critically damped steering response to a step input. It can be seen that the response tends to zero when t is large because the exponential outside the large bracket tends to zero. When $t=0$, the normalised yaw rate is zero as expected since the function simplifies to $-1+1$. The constants used in the expression are

$$\begin{aligned}r_\infty &= \frac{C_2 \partial}{k} \\ k &= \left(N_\beta + \frac{Y_\beta N_R}{mV} - \frac{N_\beta Y_R}{mV} \right) \\ \omega_n &= \sqrt{\frac{k}{I}}\end{aligned}$$

In general, engineers try to develop cars with a critically damped steering response.

For the third case, overdamped car, the solution shown below is the sum of the homogeneous solution and the particular integral. Since $\zeta^2 - 1$ is positive, its square root is a real number, and the solution is as follows:

$$\begin{aligned}I\ddot{r} - c\dot{r} + kr &= C_2 \delta \\ r &= Ae^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t} + Be^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t} + r_0\end{aligned}$$

Using the initial conditions,

$$t = 0 \rightarrow r = 0 \text{ and } t = 0 \rightarrow \dot{r} = \frac{C_f \partial a}{I}$$

Putting $t=0$, constants A and B can be calculated.

$$0 = A + B + r_\infty$$

$$B = -(A + r_\infty)$$

Introducing $f = (-\zeta - \sqrt{\zeta^2 - 1})\omega_n$, $g = (-\zeta + \sqrt{\zeta^2 - 1})\omega_n$ and $h = f/g$

$$\dot{r} = Ag\omega_n e^{g\omega_n t} + Bf\omega_n e^{f\omega_n t}$$

$$\dot{r}_0 = Ag + Bf$$

$$\dot{r}_0 = A(g - f) - r_\infty f$$

$$A = \frac{r_\infty f + \dot{r}_0}{(g - f)}$$

We have

$$\frac{r}{r_\infty} = Ae^{g\omega_n t} + Be^{f\omega_n t} + 1$$

Where

$$r_\infty = \frac{C_2 \partial}{k}$$

$$\dot{r}_0 = -\frac{C_f \partial a}{r_\infty I}$$

$$f = (-\zeta - \sqrt{\zeta^2 - 1})\omega_n$$

$$g = (-\zeta + \sqrt{\zeta^2 - 1})\omega_n$$

$$\omega_n = \sqrt{\frac{k}{I}}$$

$$A = \frac{\dot{r}_0 + r_\infty f}{(g_n - f)}$$

$$B = -(A + r_\infty)$$

Figure 4.22 shows the transient yaw rate response for understeer (underdamped), neutral (critically damped) and oversteer (overdamped) cases. The response attitude is similar. The best response, as stated in case 2, is that of a neutral vehicle. For the understeer there is a response overshoot due to the sub-critical

damping, getting worse with increasing speed. For oversteer, the damping is good but the response takes a long time to reach equilibrium. This is because an oversteering vehicle requires a smaller steer input for a given steady state, and this smaller input gives a smaller initial response. This slowness for oversteering vehicles to reach their steady state is one of the reasons why they are more difficult to drive.

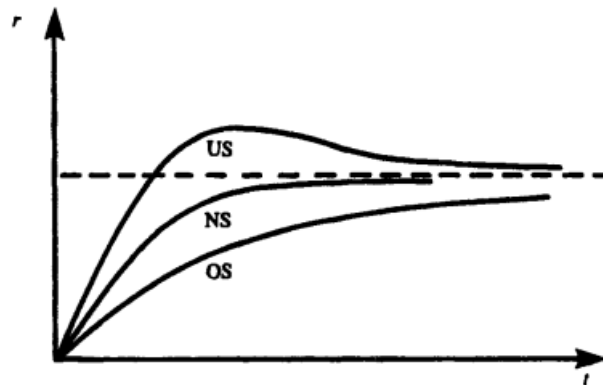


Figure 4.22. Yaw speed v time for steer input.[21]

4.8. Neutral steering car with critically damped steering.

As mentioned above, a neutral car with a zero understeer gradient is the optimal car for cornering. Let's consider the understeer gradient equation.

$$US = \frac{m}{l} \left(\frac{a}{C_R} - \frac{b}{C_F} \right)$$

This equation is zero when $bC_R = aC_F$. Considering now the steering system damping ratio equation, zeta, we have

$$\zeta = \frac{c}{c_c}$$

$$\zeta = \frac{-\left(N_R + \frac{IY_\beta}{mV}\right)}{2I\sqrt{\frac{\left(N_\beta + \frac{Y_\beta N_R}{mV} - \frac{N_\beta Y_R}{mV}\right)}{I}}}$$

The objective is to achieve an undesteer gradient of zero and a zeta value of one. The way to achieve this is to work towards a neutral steering car using a, b, C_R and C_F , and then adjust only the mass and moment of inertia to achieve a zeta value of one. In the zeta equation, taking into account that N_β and Y_R must be zero for a neutral steering car,

$$\zeta = \frac{-\left(N_R + \frac{IY_\beta}{mV}\right)}{2I\sqrt{\frac{Y_\beta N_R}{ImV}}}$$

Now we adapt this equation knowing that N_R is dependent on the velocity.

$$\zeta = \frac{-\left(\frac{N_R}{V} + \frac{IY_\beta}{mV}\right)}{2I\sqrt{\frac{Y_\beta N_R}{ImV^2}}} = \frac{-\left(N_R + \frac{IY_\beta}{m}\right)}{2I\sqrt{\frac{Y_\beta N_R}{Im}}}$$

By squaring the numerator and the denominator and substituting $\Psi = \frac{I}{m}$,

$$\zeta = \frac{(N_R + \Psi Y_\beta)^2}{4Y_\beta \Psi N_R}$$

For zeta to be one, the top part must be equal to the bottom part.

$$\begin{aligned} (N_R + \Psi Y_\beta)^2 &= 4Y_\beta \Psi N_R \\ \Psi^2 Y_\beta^2 + 2N_R \Psi Y_\beta - 4Y_\beta N_R &= 0 \\ \Psi^2 Y_\beta^2 + (2N_R Y_\beta - 4Y_\beta N_R)\Psi + N_R^2 &= 0 \\ \Psi^2 Y_\beta^2 - 2N_R \Psi Y_\beta + N_R^2 &= 0 \\ \Psi &= \frac{2N_R Y_\beta \pm \sqrt{(2N_R Y_\beta)^2 - 4Y_\beta^2 N_R^2}}{2Y_\beta^2} \\ \Psi &= \frac{2N_R Y_\beta}{2Y_\beta^2} \\ \Psi &= \frac{N_R}{Y_\beta} \end{aligned}$$

If the ratio $\frac{I}{m}$ is equal to $\frac{N_R}{Y_\beta}$, we will also have a critically damped steering response for a vehicle that is neutral steering. Actually, it is difficult to achieve the desired ratio, because in high performance cars, the aim is to reduce the weight,

so once the vehicle concept is designed, it is difficult to modify the polar moment of inertia.

4.9. Steady-state vehicle response to control and stability

According to Dixon, [21], control is action by the driver intended to influence the motion of the car. The driver seeks to exercise control over the vector velocity of the vehicle, and hence over its path. Stability refers to the unwillingness of a car to be deflected from its existing path – usually a desirable trait, in moderation.

From the fundamental equations of motion of a vehicle,

$$mV(\dot{r} + \dot{\beta}) = Y_{\beta}\beta + Y_R r + Y_{\delta}\delta$$

$$I\dot{r} = N_{\beta}\beta + N_R r + N_{\delta}\delta$$

In this section we will consider the separate issue of steady state response. We define the steady-state response as a vehicle in which the driver has made a steering demand, and all the transients have decayed away so $\dot{r}, \dot{\beta}, \dot{v}$ are zero.

[11]

$$mVr = Y_{\beta}\beta + Y_R r + Y_{\delta}\delta$$

$$N_{\beta}\beta + N_R r + N_{\delta}\delta = 0$$

These two equations relate the steering input δ to the yaw rate r through the derivative terms, velocity v and side slip β . Two control responses are used, the curvature response and the yaw rate response.

4.9.1. Curvature response

Relate the radius of the curve to the steering demand. Ideally it will be a linear relationship and twice the steering demand will be a corner of half the radius. The radius of interest will be:

$$\text{Curvature response} = \frac{1/R}{\delta} = \frac{1}{R\delta}$$

To make use of this ratio, we would first need to find an expression to which it equates. Once the value of this expression is known, we can then use the value obtained to determine R from any given δ or visa versa. [11].

$$mVr = Y_\beta\beta + Y_Rr + Y_\delta$$

$$N_\beta\beta + N_Rr + N_\delta\delta = 0$$

In steady-state cornering, $r = V/R$,

$$m\frac{V^2}{R} = Y_\beta\beta + Y_R\frac{V}{R} + Y_\delta\delta$$

$$N_\beta\beta + N_R\frac{V}{R} + N_\delta\delta = 0$$

$$-Y_\delta\delta = Y_\beta\beta + \frac{1}{R}(VY_R - mV^2)$$

$$-N_\delta\delta = N_\beta\beta + \left(\frac{1}{R}\right)VN_R$$

Eliminating β and rearranging,

$$\beta = -\frac{N_\delta}{N_\beta}\delta - \left(\frac{1}{R}\right)V\frac{N_R}{N_\beta}$$

Substituting

$$-Y_\delta\delta = Y_\beta\left\{-\frac{N_\delta}{N_\beta}\delta - \left(\frac{1}{R}\right)V\frac{N_R}{N_\beta}\right\} + \left(\frac{1}{R}\right)(VY_R - mV^2)$$

$$-Y_\delta\delta + Y_\beta\frac{N_\delta}{N_\beta}\delta = -Y_\beta\left(\frac{1}{R}\right)V\frac{N_R}{N_\beta} + \left(\frac{1}{R}\right)(VY_R - mV^2)$$

$$\delta\left\{Y_\beta\frac{N_\delta}{N_\beta} - Y_\delta\right\} = \left(\frac{1}{R}\right)\left((VY_R - mV^2) - Y_\beta V\frac{N_R}{N_\beta}\right)$$

$$\delta\{Y_\beta N_\delta - Y_\delta N_\beta\} = \left(\frac{1}{R}\right)(N_\beta(VY_R - mV^2) - Y_\beta V N_R)$$

Therefore,

$$\frac{\left(\frac{1}{R}\right)}{\delta} = \frac{\{Y_\beta N_\delta - Y_\delta N_\beta\}}{(N_\beta(VY_R - mV^2) - Y_\beta V N_R)} = \frac{\{Y_\beta N_\delta - Y_\delta N_\beta\}}{V(N_\beta Y_R - mV N_\beta - Y_\beta N_R)}$$

$$\frac{\left(\frac{1}{R}\right)}{\delta} = \frac{1}{\frac{V(N_{\beta}Y_R - Y_{\beta}N_R)}{\{Y_{\beta}N_{\delta} - Y_{\delta}N_{\beta}\}} - \frac{mV^2N_{\beta}}{\{Y_{\beta}N_{\delta} - Y_{\delta}N_{\beta}\}}}$$

Substituting for the derivative expressions in the first term of the denominator yields

$$\frac{V(N_{\beta}Y_R - Y_{\beta}N_R)}{\{Y_{\beta}N_{\delta} - Y_{\delta}N_{\beta}\}} = a + b = l$$

Thus,

$$\frac{\left(\frac{1}{R}\right)}{\delta} = \frac{1}{1 - \frac{mV^2N_{\beta}}{\{Y_{\beta}N_{\delta} - Y_{\delta}N_{\beta}\}}} = \frac{\left(\frac{1}{l}\right)}{1 + \left(\frac{1}{l}\right)\left\{\frac{mN_{\beta}}{(Y_{\delta}N_{\beta} - Y_{\beta}N_{\delta})}\right\}V^2}$$

Can be simplified in

$$\frac{\left(\frac{1}{R}\right)}{\delta} = \frac{\left(\frac{1}{l}\right)}{1 + KV^2} \quad (4.5)$$

Where K is the stability factor, given by

$$K = \left\{\frac{mN_{\beta}}{(Y_{\delta}N_{\beta} - Y_{\beta}N_{\delta})}\right\}\left\{\frac{1}{l}\right\} \quad (4.6)$$

4.9.2. Yaw rate response

Is the relation between the yaw velocity and the steering demand

$$\text{Yaw velocity response} = \frac{r}{\delta}$$

Multiplying the equation 4.5 by V ,

$$\frac{\left(\frac{V}{R}\right)}{\delta} = \frac{\left(\frac{V}{l}\right)}{1 + KV^2}$$

Taking into account that $r = V/R$

$$\frac{r}{\delta} = \frac{\left(\frac{V}{l}\right)}{1 + KV^2} \quad (4.7)$$

4.9.3. Stability factor K

K dominates the response. In equations 4.5 and 4.7, if the K value is greater than zero, the denominator will always be positive, and the response in both the yaw and curvature cases will decrease as the velocity increases. If K is negative, the denominator may be too small or zero, causing the response to tend to infinity. K takes its sign from the numerator, because of the denominator.

$$Y_{\delta}N_{\beta} - Y_{\beta}N$$

Substituting for the vehicle parameters corresponding to each derivative term,

$$\begin{aligned} &= -C_F(aC_F - bC_R) - (C_F + C_R)(-aC_F) \\ &= -aC_F^2 + bC_FC_R + aC_F^2 + aC_FC_R \\ &= lC_FC_R \end{aligned}$$

According to the sign criteria we have taken, C_F and C_R are negative, so the denominator is always positive, leaving the numerator to dictate the sign. The numerator takes its sign from the derivative term N_{β} , given by

$$N_{\beta} = aC_F - bC_R$$

This has the same sign as the understeer gradient, so it is a measure of understeer. There are 3 possible cases for different values of K, shown below.

4.9.4. Characteristic speed and positive K

To begin with, the effect of K on the curvature response, looking at equation 4.5, if K is positive, as the velocity increases, the denominator becomes larger, and the right-hand side smaller. Therefore, for a given curve radius, δ must increase with speed, and we will have an understeering car. As speed increases, the denominator increases slightly, and the car will become increasingly understeering with increasing speed. This can be seen in figure 4.23.

On the other hand, the effect of K on the yaw response comes from equation 4.7. As V appears above and below, we see that for very small values of V, the denominator is approximately one and the curvature response increases linearly with V. The bottom row, being squared, will dominate as V increases, and hence the yaw rate response will decrease. In between, there will be a local maxima. The speed at which the maxima occurs is called the characteristic speed, given by,

$$V_{char} = \sqrt{\frac{-N_{\beta}(VY_r) + Y_{\beta}(VN_r)}{N_{\beta}m}} = \sqrt{\frac{1}{K}}$$

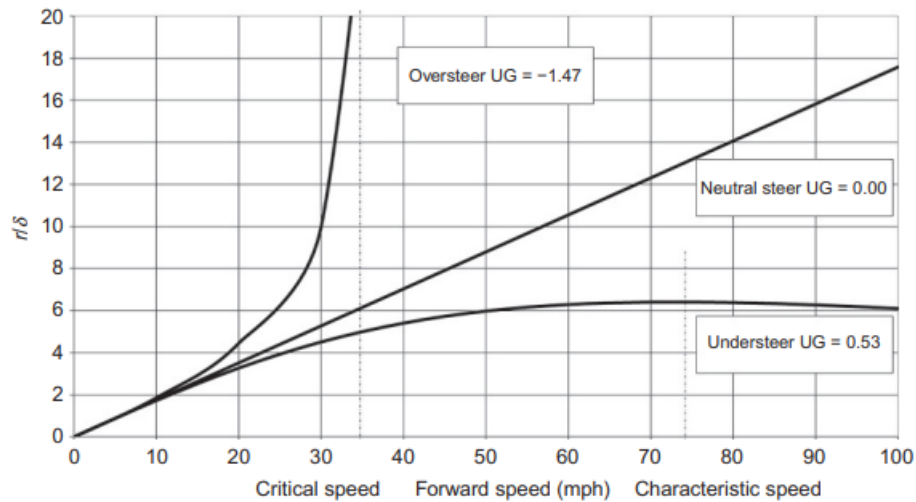


Figure 4.23. Yaw rate response for three values of understeer gradient. [11]

4.9.5. The neutral steering car and zero K

If K is zero, curvature response is,

$$\frac{\left(\frac{1}{R}\right)}{\delta} = \frac{1}{l}; \quad \delta = \frac{L}{R}$$

And yaw response,

$$\frac{r}{\delta} = \frac{V}{l}; \quad R = \frac{V}{r}$$

Both are with Ackermann steering geometry and indicate that the radius of the curve is independent of the speed, so $K=0$ is a neutral vehicle.

4.9.6. Negative K and critical speed

Denominator of Eqs 4.5 and 4.7 is given by

$$1 + KV^2$$

If K is negative, the denominator becomes smaller as the velocity increases, reaching zero at some point. Here, the curvature response and yaw response are infinitely large when this happens.

The velocity when this point is reached is called the critical velocity. It is an unstable behaviour, and the oversteering car has a discontinuity with speed. For an oversteering car the critical speed is obtained by equating the denominator of equations 4.5 and 4.7 to zero.

$$KV_{crit}^2 = -1; \quad V_{crit} = \sqrt{\frac{-1}{K}} \quad (3.8)$$

Where

$$K = \left\{ \frac{mN_{\beta}}{(Y_{\delta}N_{\beta} - Y_{\beta}N_{\delta})} \right\}$$

An important point to note here is that we will try to provide the pilot with a linear steering response, so oversteer of any kind is undesirable. This is in theory, however, in reality it is impossible to achieve a value of $K = 0$ (neutral steering car), so if the car is oversteering, the critical speed must be above the top speed of the vehicle. The general response of the vehicle to a steering input and its stability is shown in figure 4.24.

4.9.7. Static margin and neutral steer point

The neutral steer point is the point along the length of the vehicle at which an applied force produces non-steady-state yaw velocity (Figure 4.25). It is important in terms of aerodynamic design, e.g. when having a gust of wind, the car must not produce a steering input. In the picture the neutral steer point is at a distance d behind the front wheel. If we assume the car to be in steady state with $\dot{\theta} = 0$ and $r = 0$, so therefore, $\dot{\beta} = 0$ y $\dot{r} = 0$ then the fundamental equations of motion,

$$\begin{aligned} mV(r + \dot{\beta}) &= Y_{\beta}\beta + Y_R r + Y_{\delta}\delta \\ I\dot{r} &= N_{\beta}\beta + N_R r + N_{\delta}\delta \end{aligned}$$

Reduce to the following when the lateral force and torque generated by the lateral force are considered:

$$0 = Y_{\beta}\beta + F_0$$

And

$$0 = N_{\beta}\beta - (d - a)F_0$$

Therefore,

$$\beta = (d - a) \frac{F_0}{N_\beta}$$

And so,

$$0 = F_0 + Y_\beta \left((d - a) \frac{F_0}{N_\beta} \right)$$

$$0 = F_0 \left(1 + \frac{Y_\beta}{N_\beta} (d - a) \right)$$

$$d = a - \frac{N_\beta}{Y_\beta}$$

It can also be called NSP, where NSP is the portion of the wheelbase behind the front wheel.

$$NSP = \frac{d}{l} = \frac{a}{l} - \frac{N_\beta}{Y_\beta l}$$

The term static margin, SM is used to locate the neutral steer point.

$$SM = \frac{d - a}{l}$$

$$SM = NSP - \frac{a}{l}$$

$$SM = -\frac{N_\beta}{Y_\beta l}$$

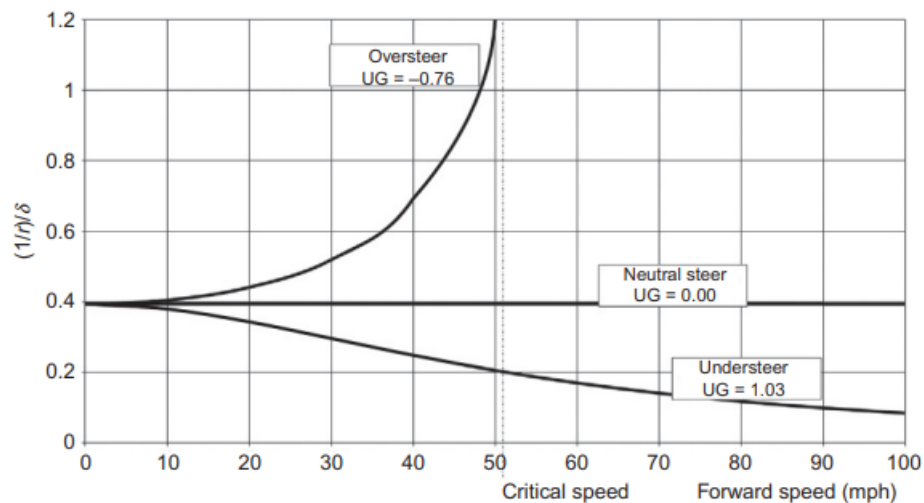


Figure 4.24. Curvature response v speed for three values of US gradient. [11]

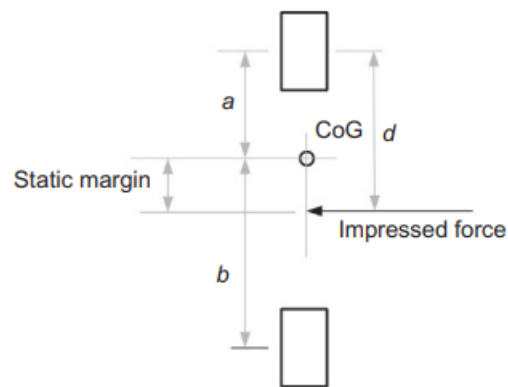


Figure 4.25. Neutral steer point. [11]

4.10. Acceleration and braking

It will start by defining the basis of acceleration. Figure 4.26 shows a vehicle at constant speed that is trying to increase its speed. In order to increase the speed, it is necessary to have an acceleration for a certain time.

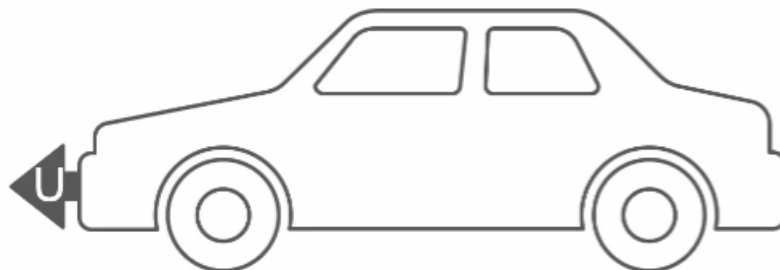


Figure 4.26. Vehicle at constant speed U. [10]

Therefore, the new speed will be:

$$V = U + at$$

To achieve the acceleration "a" a force " F_x ", which is the acceleration force, is needed.

$$F_x = \frac{T}{r}$$

Where T is the torque (Nm) and r is the radius of the wheel (m).

The wheels of the vehicle theoretically push the vehicle "backwards", but as they have grip and by the principle of action-reaction, if the wheel pushes the

ground backwards, the ground will push the wheel forwards, as shown in figure 4.27 by the force F_{RX} .

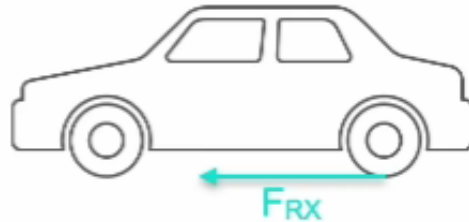


Figure 4.27. Forces on the tyre. [10]

$$F_{RX} = m * a_x \text{ or } a_x = \frac{F_{RX}}{m}$$

Linear power and angular power must also be defined.

$$\text{Linear Power} \rightarrow P = F * V$$

$$\text{Angular Power} \rightarrow P = T * \omega$$

However, traction losses must be taken into account.

$$P_R = F_{RX} * V_X$$

Simplifying:

$$a_x = \frac{P}{mV_x}$$

In addition to traction losses, there are also drag losses (D), which is the force that pushes the vehicle backwards due to aerodynamic drag.

$$F_X - D = m * a_x$$

Wheel acceleration losses must also be considered. Knowing that:

$$V = \omega * r$$

Then:

$$a_x = a_\omega * r$$

It is also known that:

$$T = I * a_\omega$$

With this in mind, the above equations can be combined:

$$a_\omega = \frac{a_x}{r}$$

And finally:

$$T = \frac{a_X * I}{r}$$

The following consideration should be added to all this:

$$T = F_X * r$$

The final acceleration will now be analysed according to the torque.

$$T = r * (m * a_X + D) + \frac{4 * a_X * I}{r};$$

$$T = r * a_X * m + r * D + \frac{4 * a_X * I}{r};$$

$$T = r * a_X * m + \frac{4 * a_X * I}{r} + r * D;$$

$$T = a_X \left(r * m + \frac{4 * I}{r} \right) + r * D;$$

$$a_X \left(r * m + \frac{4 * I}{r} \right) = T - r * D;$$

So, the linear acceleration is:

$$a_X = \frac{T - r * D}{r * m + \frac{4 * I}{r}}$$

Braking forces and braking times will be discussed below.

$$m * a_X = F_{braking} + D;$$

$$a_X = \frac{F_{braking} + D}{m};$$

$$\frac{\Delta V}{t} = \frac{F_{braking} + D}{m};$$

$$\frac{V_i - V_f}{t} = \frac{F_{braking} + D}{m};$$

If the final velocity and drag were 0:

$$\frac{V_i}{T_S} = \frac{F_{braking}}{m};$$

Where T_S is the time until the car stops.

$$T_S = \frac{V_i}{\frac{F_{braking}}{m}};$$

To conclude this chapter, the total braking distance will be analysed without taking drag into account.

$$\frac{\Delta V}{t} = \frac{F_{braking} + D}{m};$$

$$\frac{F_{braking}}{m} = \frac{dV}{dt};$$

Figure 4.28 shows that the speed decreases over time when braking, and at the same time, the distance increases, but it increases more slowly as the speed decreases.

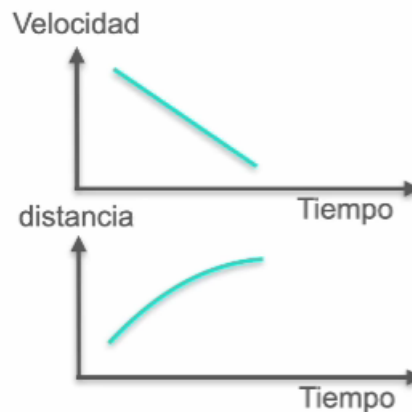


Figure 4.28. Speed v time and distance v time.[10]

$$V = \frac{dx}{dt};$$

$$dt = \frac{dx}{V};$$

Now, substituting into the above equations:

$$\frac{F_{braking}}{m} = V \frac{dV}{dx};$$

$$V dV = \frac{F_{braking}}{m} dx;$$

Integrating both elements:

$$\int_{V_i}^{V_f} V dV = \frac{F_{braking}}{m} \int_0^X dx;$$

$$\frac{1}{2} (V_i^2 - V_f^2) = \frac{F_{braking}}{m} X;$$

$$\text{If } V_f = 0;$$

$$X_S = \frac{V_i^2}{2 \frac{F_{braking}}{m}}$$

And X_s is the distance the vehicle will travel until it comes to a complete stop.

5. Braking

On typical racetracks, drivers only spend about 10% of the time braking . When they brake, brakes tend to be fully engaged and the deceleration is high. If we achieve a braking system that allow our driver a slightly higher deceleration than the other competitors, he would be able to brake later and spend less time at low speed. Even small advantages in braking performance, lead to much higher end positions when margins are small. So we conclude that improving lap time performance by improving braking performance will be difficult, but any small gain will be welcome. [11]

Passive systems are usually mandatory in racing, but on the road cars this is not the case at all. The advanced braking system (ABS) will prevent a vehicle from skidding no matter how hard the brakes are applied.

5.1. Braking Energy

Braking systems almost always involve frictional contact between one surface that is driven by the wheels and another that is held stationary with respect to a part of the vehicle. Thus, during braking, the kinetic energy of the moving vehicle is dissipated in the form of increased heat in the brake materials, which, once hot, must lose their heat to the airflow.[11]

We will estimate the energy dissipated by a 1000 kg road vehicle decelerating from 20 m/s.

$$E_{TOT} = \frac{1}{2} mV^2 = \frac{1}{2} * 1000 * 20^2 = 0.2MJ$$

If the car needed 4 seconds to stop, the rate of energy delivery is

$$\frac{0.2 * 10^6}{4} = 50000 = 50kW$$

If we now assume that the friction material of the brakes directly warmed up by braking weighs 1.5 kg per wheel and has a specific heat capacity of 500 J/kg/K, then we may estimate the temperature rise of this material to be as follows:

$$H = MC\Delta\theta$$

$$0.2 * 10^6 = 4 * 1.5 * 500 * \Delta\theta$$

$$\Delta\theta = 66.66^\circ$$

Thus, for a road car, the quantities of energy involved are not so great, and the thermal mass of the discs themselves are sufficient to absorb much of the energy. We can compare this with a racing car of mass 600 kg decelerating from 70 m/s to rest, for which the energy lost is:

$$E_{TOT} = \frac{1}{2}mV^2 = \frac{1}{2} * 600 * 70^2 = 1.47 \text{ MJ}$$

The braking time in racing cars is significantly less than in road cars, so let's say it takes 2 seconds to stop.

$$\frac{1.47 * 10^6}{2} = 735000 = 735 \text{ kW}$$

If we again assume the friction material of the brakes to be 1.5 kg per wheel and specific heat capacity 500 J/kg/K, then we may estimate the temperature rise to be as follows:

$$H = MC\Delta\theta$$

$$1.47 * 10^6 = 4 * 1.5 * 500 * \Delta\theta$$

$$\Delta\theta = 490^\circ$$

In fact, this is a very poor estimate of the temperature rise because everything happens so quickly that heat is transferred to the body of the commonly used carbon disk. What actually happens is that the surface temperature rises very rapidly up to about 850°C. When this temperature is reached, the surface is in temporary equilibrium with the generated heat flowing to the surface of the disk and from there enters the directed airflow above them. [11]

Therefore, for racing vehicles, brakes with materials that dissipate heat quickly are required. This means that from a design point of view, brakes need to operate at

the maximum possible temperature and be well ventilated, characteristics that suit carbon fibre very well, as well as saving weight.

Since it is the contact patch that produces the longitudinal deceleration force, the requirement of the brakes is that it should be able to lock all four wheels. In addition, a well-designed braking system will allow the driver to apply a braking intensity lower than the locking intensity, i.e. the brakes can operate smoothly over a range of brake pedal pressure. It is not desirable for the brakes to lock the wheels at the lowest brake pressure applied. Drivers need to be able to control the braking effort applied to the wheels smoothly and accurately.

5.2. Braking force

Braking force is the force developed at the tyre-asphalt contact as a reaction to the torque opposing the rotational movement of the tyre, and is mainly limited by the adhesion. If the limit of adhesion of the wheel-road contact is exceeded, wheel locking occurs.

Very worn tyres or a wet, muddy, snowy or icy road significantly reduce contact grip, and wheel locking occurs with less force, which results in a reduced braking force, resulting in slippage and loss of directionality of the car. [22]

The braking torques on non-traction wheels are due to the forces generated by the braking system, due to the friction of the brake pads against the brake discs. In the case of wheels with traction, the torques opposing the rotation of the wheel may be due to the engine, the braking system or a combination of both.

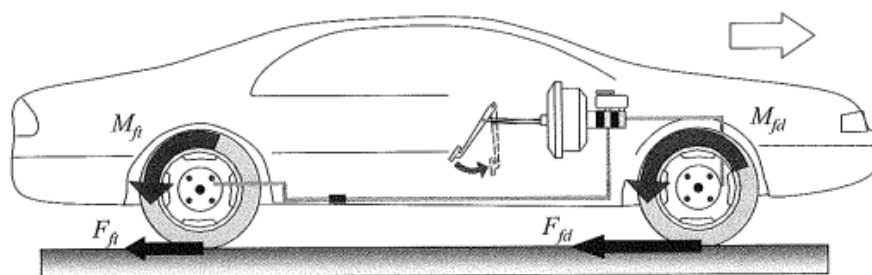


Figure 5.1. Braking torques. [7]

Pressing the brake pedal increases the pressure in the circuit, which is converted in the wheel devices into pairs M_{fd} and M_{ft} , applied in the opposite direction to

their movement. When this happens, the ground reacts with a force on the tyre, called braking force, which will try to keep the tyre rotating. In a disc brake, increasing the pressure in the brake circuit displaces "pads" which, when rubbing against the disc, generate a frictional force of value F_{f_disc} applied at a distance from the wheel axle. The braking torque will be: [7]

$$M_f = F_{f_disc} * r_{disc}$$

Where r_{disc} is the mean radius of the brake disc, where the braking force is assumed to be applied to the disc.

Using the free body diagram in figure 5.2, the relationship between the braking torque due to the braking system and the braking force on the ground will be:

$$M_f - F_f * r_c = I * \alpha$$

Where:

r_c : is the mean radius under load of the tyre, which can be considered equal to the effective radius.

I : is the moment of inertia of all masses attached to the wheel.

α : is the angular acceleration of the wheel.

F_f : is the braking force generated on the ground.

Therefore, the ground reaction torque on the wheel will be:

$$M_R = F_f * r_c$$

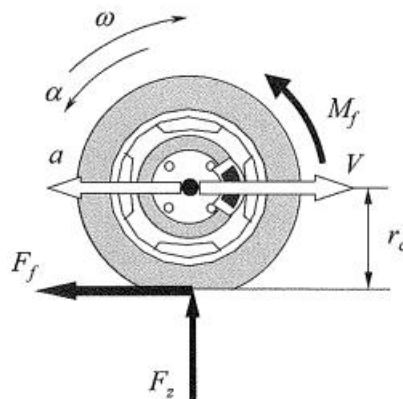


Figure 5.2. Braking force and braking torque.[7]

From the expression relating the braking torque to the braking force it can be deduced that part of the braking torque is consumed in the ground reaction, in the

braking force, and another part is consumed in braking the rotating masses attached to the wheels. The braking force will therefore have the following values:

$$F_f = \begin{cases} \frac{M_f - I * \alpha}{r_c} \leq F_{adhmax} = F_z * \mu_{max} & \text{without locking on the wheel} \\ F_z * \mu_d & \text{with the wheel locked} \end{cases}$$

From the above expression it can be deduced that the braking force is limited by the adhesion between the wheel and the asphalt. Its maximum value is obtained as long as the wheel is not blocked, this value being equal to the vertical load on the tyre times the maximum coefficient of adhesion. Wheel locking means that the braking force takes a constant value equal to the dynamic coefficient of adhesion times the vertical force on the tyre, being, in this case, lower than the maximum value which would be reached without wheel locking. [7]

5.3. Longitudinal slippage

As seen in previous chapters, due to the inertia of the car, under acceleration and especially under heavy braking, a longitudinal weight transfer occurs. Under braking, the rear axle is lightened and the front axle is overloaded. Due to this increased front axle load, the adhesive force also increases, so the braking force must be increased on the front wheel brakes. If the braking force were the same on the front and rear wheels, the rear wheels would lock.

Let's look at the rear axle locking situation. This situation occurs when the front axle has grip but the rear axle has become locked.

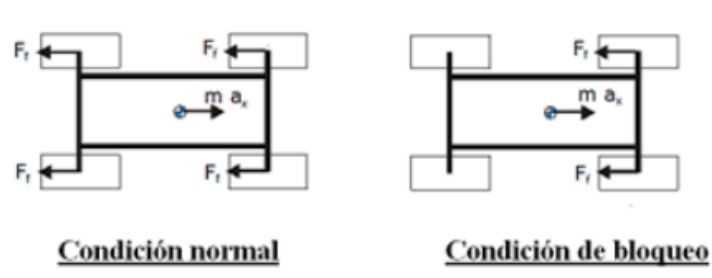
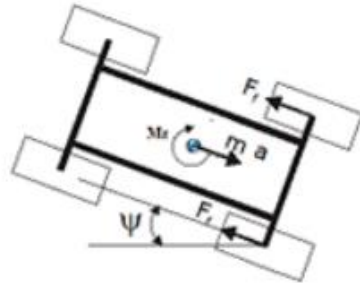


Figure 5.3. Stresses at normal condition and rear axle lock, left to right. [22]

However, when a vehicle is driving, even on a straight line, it is slightly off the horizontal line, forming an angle ψ with the horizontal. This produces a yaw moment which causes the rear of the vehicle to tend to rotate around the axis

perpendicular to the roadway passing through the centre of gravity, resulting in the so-called "directional instability".

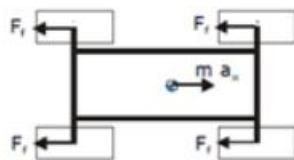


$$\frac{\partial M_z}{\partial \psi} > 0 \rightarrow \textit{instability}$$

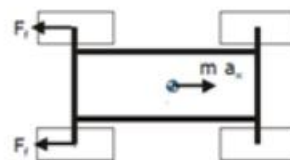
Figure 5.4. Directional instability.[22]

Due to this locking of the rear wheels, the wheels on the rear axle slip, which causes the car to spin, moving the rear of the vehicle forward to the front, which is known as a spin.

Let us now consider the opposite case, if the blockage were to occur on the front wheels. In this case, the axle that has grip with the road is the rear axle, but the front axle has become blocked.



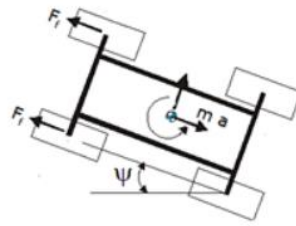
Condición normal



Condición de bloqueo

Figure 5.5. Normal and locking condition on front axle, left to right.[22]

In this case, the inertia forces applied at the centre of gravity and the friction or adhesion forces on the wheels provide a yaw moment that decreases with the lateral disturbance. [22]



$$\frac{\partial M_z}{\partial \psi} < 0 \rightarrow \text{stable} + \text{oscillation yaw}$$

Figure 5.6. Loss of directional control.[22]

The yaw angle will reach a maximum value, after which it will decrease and may change direction, producing some loss of directional control, less severe than the instability.

5.4. Transverse sliding

When cornering, the steered wheels are oriented according to the curvilinear trajectory, but due to the effect of the disturbing forces acting on the car, a lateral displacement is generated which modifies the instantaneous centre of rotation and the theoretical directional trajectory, causing the car to tend to run off the road.[23]

The most important factor influencing the drift of a car in curves is the centrifugal force ($F = m * \frac{v^2}{R}$). When the speed is too high or the curve too sharp, the centrifugal force action on the car increases and the driver has to adjust the driving speed.

Furthermore, this situation can be critical if the car's own stability limit defined by the so-called tyre grip circle is exceeded. According to this circle, when the resultant TC between the traction force T and the centrifugal force C exceeds this circle of adhesion, this causes the wheels to skid and the car to exit the turn. [22]

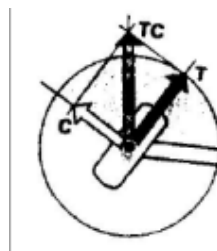


Figure 5.7. Tyre grip circle.[22]

Furthermore, it must be taken into account that the centrifugal force acting on the curves and the same inclination of the car towards the outside of the curve, generates a transverse weight transfer, i.e. the outside wheels are loaded and the inside wheels are lightened, which modifies the adherence of the wheels on both sides and also the positioning of the centre of gravity.

At the same coefficient of adhesion, the frictional force on the outside wheels is higher than on the inside wheels, because, as described above, the outside wheels are loaded more when cornering. In this situation, the resultant between the braking force and the centrifugal force can exceed the limit of the adhesion circle, and lateral sliding under braking can occur. In addition, as the inside wheels become lighter, at equal braking force, they may lock up.

A number of important points can be concluded from this:

- 1) The locking of the rear axle of a two-axle vehicle produces a high degree of directional instability of an irreversible nature.
- 2) The locking of the wheels on the front axle of a two-axle vehicle can cause loss of directional control.
- 3) The locking decreases the normal coefficient of adherence, passing to the sliding friction value, which, in the best of cases, if there were no serious alteration of the trajectory, would increase the braking distance with respect to the optimum condition, that is, μ , if the adherence is maximized

For this reason, both the design of the braking system and the driving must act in such a way that the locking of the front or rear wheels is avoided. Under sudden braking, especially in low grip conditions, lockup may occur and it is likely that the wheels on both axles will not lockup at the same time. In this case, it is less unfavorable if the blockage occurs earlier in the front wheels. For this reason, elements are added to the system that limit braking on the rear axle, so that it does not block before it does on the front axle.

5.5. Optimum distribution of braking forces

When the vehicle is stationary, its mass is distributed between the front and rear axles, with values that the vehicle design has provided. Almost all commercial

vehicles today are slightly heavier in the frontal area than in the rear. This is due to the location of the engine in this part as well as that when pulling on the front axle, the gearbox, differential, transmissions, etc. They are also on it. [24]

Thus the lower weight on the rear axle means that the design of the braking distribution is critical to avoid rear wheel lock-up. This, together with the mass transfer from the rear axle to the front axle means that not only is the rear axle lighter than the front axle, but also because of the vehicle dynamics at the rear axle, whenever the brake is applied, it is going to be unload by transferring some of that load to the front axle.

Because of this, the braking force to be applied to the front axle is not equal to that of the rear axle.

This can be shown in graphs which represent the relative deceleration during braking with respect to the axle load distribution.

In the following example we see, for a value of $\mu=0.8$, what happens to the braking distribution on each axle and its consequent locking. [22]

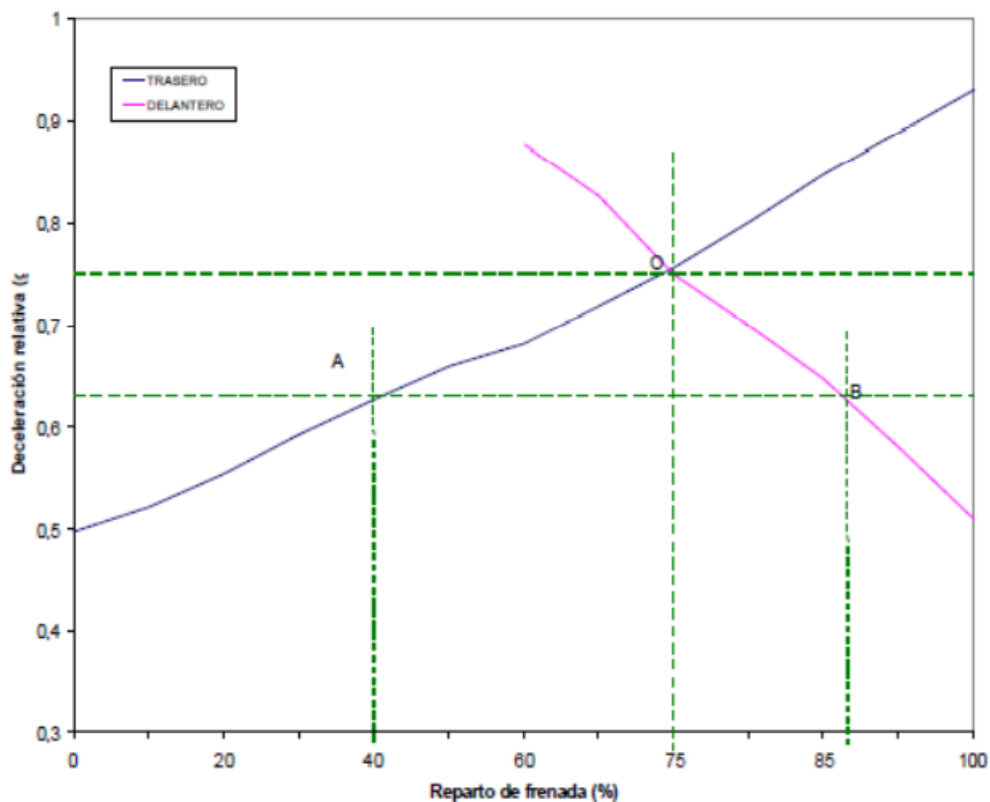


Figure 5.8. Optimal braking distribution.[22]

The rear axle is shown in blue, the front axle in pink, the brake distribution on the abscissa axis and the relative deceleration on the ordinate axis.

At the point of intersection of the two curves corresponding to the front and rear axle (point O), a braking distribution of 75% on the front axle and 25% on the rear axle would be determined. In this case we would have a relative deceleration of approximately 0.75. Thus, we would be making maximum use of the available grip. [25]

However, if we look at point B, the braking distribution would be 86% for the front axle and 14% for the rear axle, with the front axle locking earlier and achieving a maximum relative deceleration of 0.62 which is well below than the optimal one of 0.75.

Now if we look at point A, 40% would go to the front axle and 60% to the rear axle, locking the rear axle first and with a maximum relative deceleration of 0.62.

Therefore, in view of these two totally different situations, we would choose the option of point B (86% on the front axle and 14% on the rear axle) as this gives the same deceleration as point A, but knowing that the front axle will be the first to lock, which, as mentioned above, is preferable.

Another resource used for the calculation of the optimum braking distribution are the so-called equiadherence curves.

These curves graphically represent the values of the braking forces at the front and rear axle under the condition that the wheels on both axles achieve maximum adhesion at the same time. [22]

The equiadherence curve is thus the geometric locus of the pairs of values which, for a given vehicle load distribution situation, achieve the maximum braking force simultaneously on both axles. There will be as many equal adhesion curves as there are possible load distributions of the vehicle.

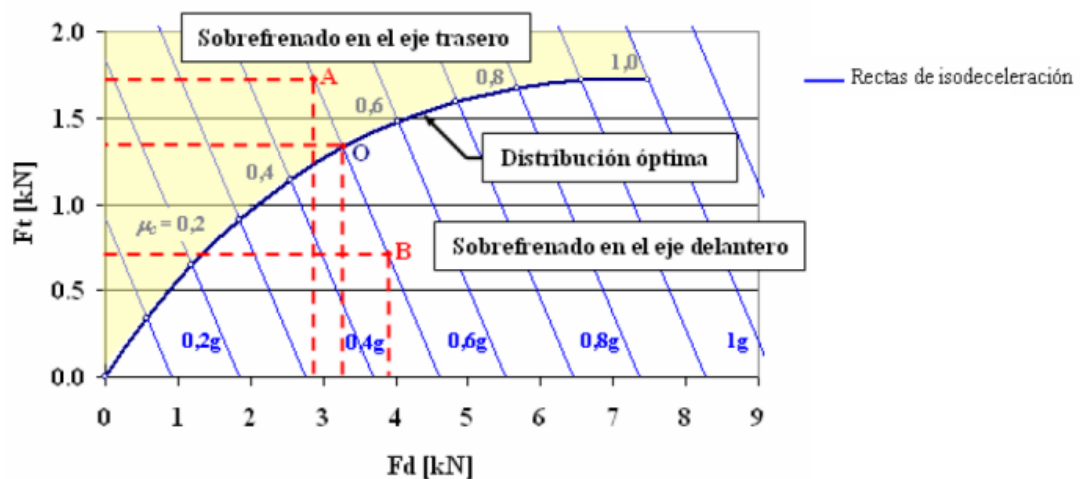


Figure 5.9. Equiadherence curve.[22]

The abscissa axis shows the front braking forces and the ordinate axis shows the rear braking forces, both in kN.

This figure shows three possible situations of brake force distribution between the front and rear axles. In all three cases the same braking deceleration is obtained. If the adhesion between the tyre and the road is at point O, this indicates the optimum distribution of the maximum braking forces. Under these conditions, the distribution corresponding to point A shows that the braking force on the rear axle is higher than the maximum permissible for that adhesion, while the braking force on the rear axle is lower, so that, the rear wheels would lock earlier than the front wheels in this situation.

If the distribution is that corresponding to point B, it would be the front axle that would lock before, since the braking force on that axle is higher than the maximum under the given conditions of adhesion.

From this analysis it can be deduced that the equiadherence curve marks two zones differentiated, the one between it and the braking force axis on the rear axle in which there is overbraking in the rear axle, and the one between it and the braking axle on the front axle where overbraking occurs on the front axle.

Taking into account all the considerations made so far, that locking the rear axle produces directional instability while locking the front axle causes a certain loss of directional control (less dangerous situation), the braking system must provide at all times distribution values located below the equiadherence curve.

5.6. Brake Systems

Braking systems can be divided into four basic areas:

- 1) Suitable mechanical components
- 2) Hydraulic system
- 3) Adequate cooling
- 4) Brake distribution (adjustable)

The master cylinder reservoir must be located so that "g" forces cannot expose the feed holes that would allow air to enter the master cylinder. The reservoir capacity must be sufficient to fill all caliper pistons with "worn-out" pads (bottomed). Check this every time the master cylinder type or calipers are changed. The master cylinder push rod should be adjusted so that the piston returns fully when the pedal is released. This allows the master cylinder to be refilled from the reservoir between brake applications. Before the system is filled, the master cylinder also can be checked for full stroke without pedal bottoming. On dual master cylinder systems equipped with a balance bar, check for bindings that tend to shift the brake balance. [3]

Whenever possible, flexible brake lines should be replaced with metal braided hoses to minimize volume changes due to hose expansion. Hose expansion is a possible cause of a soft or low pedal. All rigid lines must be steel with properly double-flared ends. In the brake line routing we should avoid loops that trap air and make bleeding difficult. Brake lines should be routed away from exhaust systems and other hot areas. Another cause of soft pedals is caliper flex, which is visible on many of the newer lightweight production calipers. While aluminum brake calipers reduce the unsprung weight, steel brake calipers of the same design are stiffer. To help smooth brake modulation, check that the calipers are working freely. It is helpful to polish up the leading edge of the opening in the caliper housing and the leading edge of the brake shoes to a smooth finish and apply a minimal amount of lubricant to reduce friction between the caliper housing and the brake pad. [3]

Once a component of the brake system has been replaced, calipers, hoses, pump, etc., it is necessary to remove the air trapped in the system. Air causes lower compressive strength, reduces thermal conductivity, can cause oxidation, lower viscosity... what many describe as spongy feel. There are other possible causes of this type of feel: excessive pad wear, bad servo, hoses, etc. [26]

Due to the extreme braking temperatures encountered in competition use DOT-approved 550° boiling point fluid. Regular fluid changes to avoid possible vapour lock under high temperature are recommended because of the affinity of glycol-based hydraulic fluid for water. Care should be taken when pouring the fluid into the master cylinder to prevent the formation of small bubbles which can become trapped in the system. Silicone fluid should be used only if the system has silicone-compatible seals. Silicone-based fluid should not be mixed with glycol-based fluid. [3]

On road cars (with and without ABS) the brake circuit is divided for safety reasons between left front and right rear (channel 1), and vice versa, right front with left rear (channel 2). We start with the rear of the same channel and then the front. The brake pump and servo (if fitted) is located next to the driver's seat on the left, so the bleed order would be:

1. Rear right
2. Front left.
3. Rear left
4. Front right

If the brake pump/servo is located next to the driver's seat on the right hand side, the order would be:

1. Rear left
2. Front Right
3. Rear Right
4. Front left

On racing vehicles the order will depend, if the arrangement of the ducts has not been modified, the above criterion is valid. Most of them may have independent

pumps for each axis or the channels are not crossed, so the order will depend on the connection. [26]

Adequate air flow for the rotors and calipers need to be provided. Air need to be ducted to the eye of the rotor and, optionally, air blast can be directed on the caliper if fluid boiling (spongy pedal) is a problem. As the the front brakes do maximum of the work, cooling them is really important.

Brake cooling air must be taken from a high-pressure location, for example, the front of an air dam (chin spoiler). To be effective, ducting should use 7.62 cm minimum diameter hose unless the duct is very short. [3]

Proper brake balance is extraordinarily critical as said withinside the preceding chapter. The car will now no longer gain the most braking deceleration until all 4 tires are delivered to the friction peak simultaneously. An improper brake balance will cause one end or the alternative to lock up first and this end of the automobile will lose cornering traction on corner entry. Proper brake balance is a function of the masses at the wheels, which is in turn a function of the deceleration. There are two common ways to modify the brake balance. The first kind is most approximate for systems that use a production-type dual piston master cylinder. This amounts to fitting a brake pressure regulator in line with whichever end of the automobile has a tendency to lock up first. The other approach is to apply a dual master cylinder set-up with an adjustable balance beam splitting the pedal pressure among the two cylinders. [3]

As mentioned above, it is preferable that the axle that locks first is the front axle. Once the rider has become accustomed to this feature, the brakes can be progressively adjusted until the rear axle lock is approached. The direction of adjustment should be considered for the system fitted to the vehicle. Approaches can be made for correct brake distribution. First, a maximum deceleration must be assumed to calculate the approximate mass on the wheels during braking. For vehicles that have to race on wet and dry tracks, the calculations must be done twice to find the required range of adjustment.

$$W_{1\text{ braked}} = W_1 + W(a/g)(h/l)$$

Where:

$W_{1\ braked}$ = weight on front axle during a stop of (a/g) deceleration.

W_1 = weight on front axle with no braking but with any aero effects at the speed of interest.

W = gross weight of the vehicle.

(a/g) = deceleration of the vehicle in "g" units.

h = height of the vehicle center of gravity above ground.

l = wheelbase.

The load on the rear track is given by:

$$W_{2\ braked} = W_2 - W(a/g)(h/l)$$

Where $W_{2\ braked}$ is the weight on rear axle during a stop of (a/g) deceleration.

Looking at these equations, it follows that at very low decelerations, the weight on the wheels is approximately equal to the weight distribution. For high decelerations, the second term of the equation becomes significant, therefore, to minimise the effect of the second term, a low CG and a long wheelbase are needed.

Next, it is necessary to have data on the coefficient of friction under braking. This coefficient (μ) decreases with increasing load (similar to cornering performance). Because of this "load sensitivity", the maximum μ will be higher at the rear axle.

Once the μ under load conditions is known, the braking force $F = W_{braked} * \mu_{effective}$ can be calculated.

The braking force for straight-line braking will be split approximately evenly left and right so the weight and braking force on each tire will be half the total for its axle. To generate this force at the wheel, the following formula may be used to calculate the required brake line pressure. [3]

$$p = \frac{F_x * (R_l/r)}{A_c * \mu_{pad}}$$

Where p = Brake line pressure.

F_x = required longitudinal force from a wheel.

(R_l/r) = loaded radius of the tire/brake caliper radius.

A_c = total caliper piston area.

μ_{pad} = coefficient of friction of brake pad (typical range = 0.3 to 0.45)

To generate this pressure, there will be a certain pedal force, as increased by pedal ratio, applied to the area of the master cylinder piston. Assuming that the system is not boosted, a master cylinder area is known, and there is a desired pedal force, the following formula will allow calculation of the pedal ratio for single master cylinder systems. If a dual cylinder system is used, the balance bar ratio must also be taken into effect. [3]

$$R_{pedal} = p * \frac{A_{master}}{F_{foot}}$$

Where R_{pedal} = distance (foot to pedal pivot)/distance (pushrod to pedal pivot); this number will be greater than 1.

A_{master} = area of master cylinder.

F_{foot} = pedal force.

Once this calculation is made, check that the pedal ratio chosen will allow the master cylinder to travel through its fully stroke. It may be necessary to cycle through the calculations several times for a new design to balance the various piston areas and to achieve a reasonable pedal force. In the calculations, it was assumed that there is no loss in the hydraulic system. [3]

4.7. Braking systems on a Formula 1 car

When a driver presses on the brake pedal in a Formula 1 car, the force he is applying to the brake pedal is applied to two master cylinders. One cylinder is a fluid that is linked to the front brake system and the other cylinder is linked to the back brake system. That pressure in that cylinder goes through a brake line and onto the calliper, onto the brake disk itself.

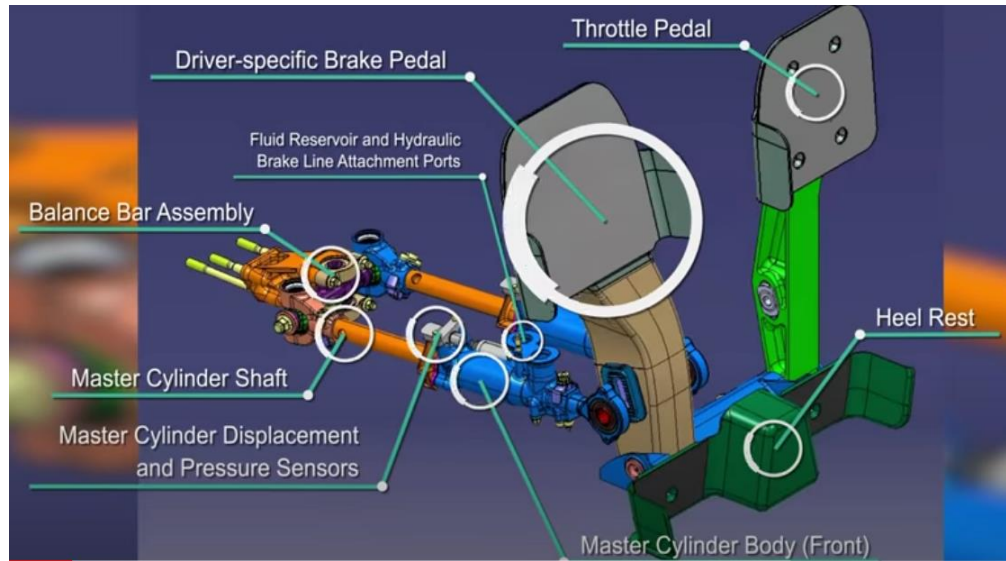


Figure 5.10. Braking system on a Formula One car.[27]

On the back of the car, the system is more complicated than that because Formula 1 cars have Energy Recovery Systems, and teams have to deal with those to get the brake balance right. [27]

On road cars, when you press the brake pedal, vehicles have an assist system, a servo-assistant that multiplies the pressure the driver is applying into the master cylinder. On a Formula 1 car, it is not possible, so the driver has to provide all that force, which is a huge amount of effort that the driver has to make. In order to do that, what happens is they are actually using the deceleration of the car and their own bodyweight sliding in the car to get the amount of pressure that they need. Therefore, the driver is not only having to press the brake pedal really hard, he is also having to withstand the huge “g” forces (around 5 g).

If we look to a brake disk, when they are braking at maximum brake temperature, on a high-duty circuit, temperatures could be up to 1000°C on one of this carbon disks. As the car goes down the straight, the air is coming through the brake duct, and temperatures at the end of the straight could be as low as 200°C. The disk itself has up to 1000 holes. It is important to understand why handling the brake temperatures is hard, if the disk gets too hot, for example, if it achieve 1000°C at the end of the straight, what happens it we start seeing fade. Equally, if the discs are too cold, something might happen at the end of a straight, then we do not get

that initial braking that teams are looking for, so they have to manage the temperatures of the discs really well. They do that with the flow that is coming through the brake ducts. That flow is fed through the duct and through the upright onto the disk, it is also available to cool the calliper and the pads that are inside the calliper. All of that has to be kept at the right temperatures to get the peak braking performance. It is also really important how to deal with the flow that comes out of the disc, how that feeds over the wheelrim and around the tyre is really important for the tyre performance, because teams need to keep those in the right operating window.

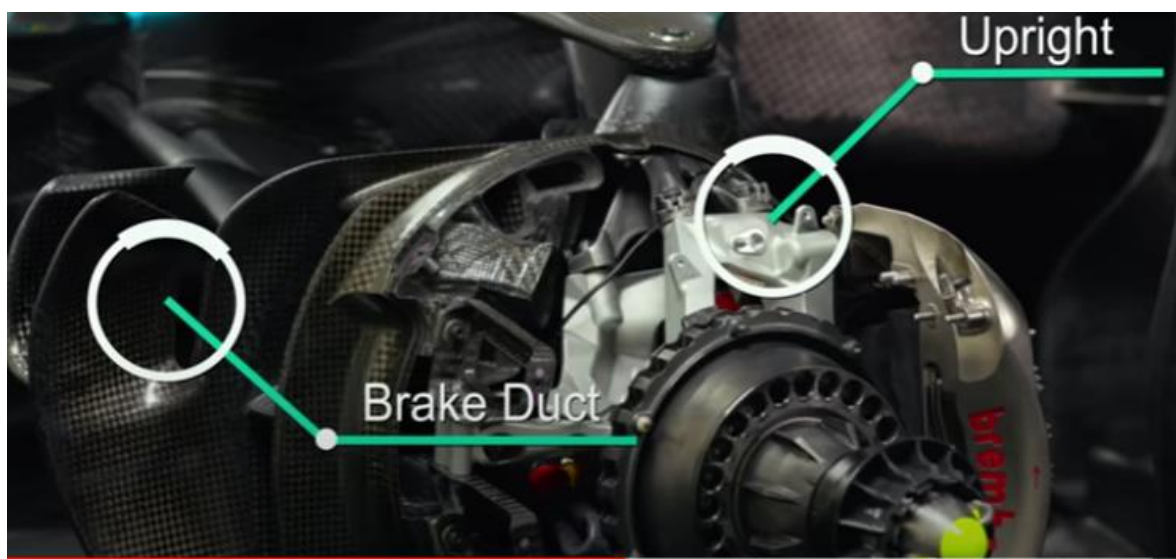


Figure 5.11. Flow on a brake disk.[27]

Drivers can vary the brake distribution between the front and the rear axle, and this changes can be made inside the car.

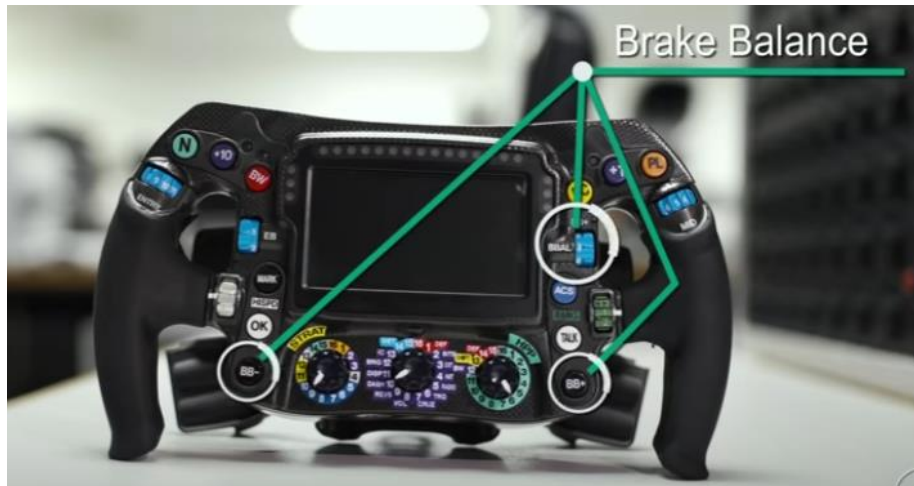


Figure 5.12. Brake balance.[27]

When people talk about brake migration, which is the same as balance migration, as the driver presses on the brake pedal, vehicles also alter the balance front to rear using the Energy Recovery Systems to get an optimal braking performance

6. Steering

5.1. Steering Geometry

The kingpin in a solid front axle is the steering pivot. In more recent independent suspensions, the kingpin is replaced by 2 or more ball joints which define the steering axis. This axis is not centred with the tyre contact patch, i.e. it is not vertical. From a front view, the angle is called kingpin inclination and the separation between the steering axis and the centre of the tyre tread measured on the ground is called scrub. The distance from the kingpin axis to the centre plane of the wheel, measured horizontally at axle height, is the spindle length. From a side view, the kingpin angle is called caster angle, if the kingpin axis does not pass through the centre of the wheel, then side view kingpin is present. The distance measured above the ground between the steering axis to the centre of the tyre tread is the trail.

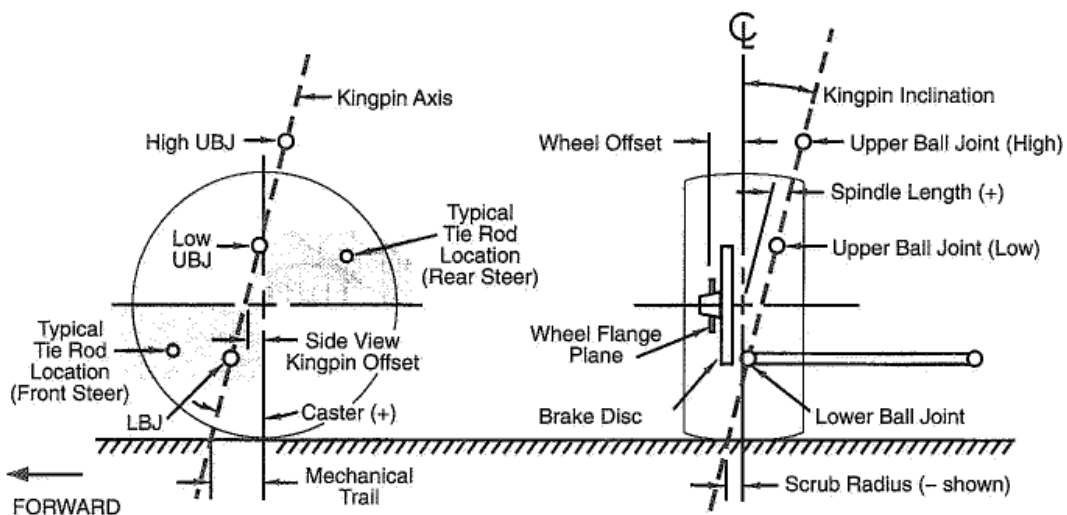


Figure 6.1. Kingpin geometry.[3]

Kingpin inclination, spindle length and scrubbing are often a compromise between packaging and performance requirements. The following factors are taken into account: [3]

1. With a positive spindle length (virtually all cars have a positive spindle length as shown in figure 6.1) the car will tend to lift when the wheels are

steered away from center. The more kingpin inclination, the more the car will lift when the front wheels are turned. This effect always lifts the car, regardless of the direction in which the wheels are turned, except when the kingpin inclination coincides with the vertical. For a given kingpin inclination, a longer positive spindle length will increase the amount of lift with steer.

2. The effect of kingpin inclination and spindle length on front axle lift tendency, in itself, serves to centre the steering at low speeds.
3. Kingpin inclination affects the steer-camber characteristics. When a wheel is steered, it will lean out at the top, toward positive camber, if the kingpin is inclined in the normal direction (toward the center of the car at the upper end). Positive camber results for both left and right hand steer. The amount of this effect is small, but significant if the truck includes tight turns. [3]
4. When driving on a bumpy road, the rolling radius is constantly varying, which results in changes in the rotational speed of the wheel. This triggers longitudinal forces on the wheel centre. The reaction of these forces will introduce kickback into the steering in proportion to the spindle length.
5. The scrub radius shown in Figure 6.1 is negative, as used on front-wheel drive vehicles. Braking and traction forces introduce steer torques proportional to the scrub radius. If these braking and traction forces are different between the left and right wheels, the rider will feel a steering torque.
6. For front-wheel drive vehicles, a negative scrub radius has two strong stabilising effects:

If one drive wheel loses traction, the opposite wheel will tend to diverge outwards by an amount determined by the steer compliance in the system. This will help to keep the car in a straight line, even when the tractive force is not equal on both sides.

Second, with good reverse efficiency the driver's hands never truly fix the steering wheel. In this case the steering wheel may be turned by the effect of uneven longitudinal tractive forces, increasing the stabilizing effect of the negative scrub radius. Under braking the same is true. Negative scrub radius tends to keep the car traveling straight even when the braking force is not equal on the left and right side front tires

With mechanical trail, shown in figure 6.1, the tyre print is kept behind the steering axis in side view. More trail implies that the tire side force has a larger moment arm to act on the kingpin axis. This produces a greater self-centering effect and is the main source of self-centering moment on the kingpin axis. The following considerations should be taken into account when choosing the caster angle and trail:

1. More trail will give higher steering force.
2. Caster angle, like kingpin inclination makes the wheel go up and down with steer. In contrast to kingpin inclination, the effect is opposite from side to side. With symmetrical geometry, when turning left, the car will tend to rotate to the right, causing a diagonal weight transfer. In this case, more load will be carried on the LF-RR diagonal, an oversteer effect in a left-hand turn. This diagonal weight transfer will be greater if the springs used are stiffer. The distance that each wheel is raised or lowered will be constant but the weight jacking and chassis roll angle are functions of the front and rear roll stiffness.
3. Caster angle affects steer camber, but unlike kingpin inclination, the effect is favourable. With positive caster angle the outside wheel will camber in a negative direction (the top of the wheel pointing towards the centre of the car) while the inside wheel cambers in a positive direction, again leaning into the turn.
4. As mentioned, the tyres have a pneumatic trail which effectively adds to the mechanical trail. This effect of the tyre is not linear with the lateral force and affects the steering torque and the driver's feeling. The fact that the pneumatic trail approaches zero when the tyre is reaching the limit will imply less self-centering moment and may be a signal to the driver that the tyre is about to lose grip.

5.2. Ackerman

Ackerman's principle states that when a vehicle is cornering, the axles of all the wheels must meet at a point, the instantaneous centre of rotation. The inner wheel spindle must rotate at a greater angle than the outer wheel, so a divergence of the

front wheels is required when cornering to avoid tyre wear and prevent slippage. With the aforementioned mechanism, we achieve an optimal steering geometry. To follow this principle, the turning angle of the inner wheel is made larger than the outer wheel, i.e. as shown in the figure 6.2. [28]

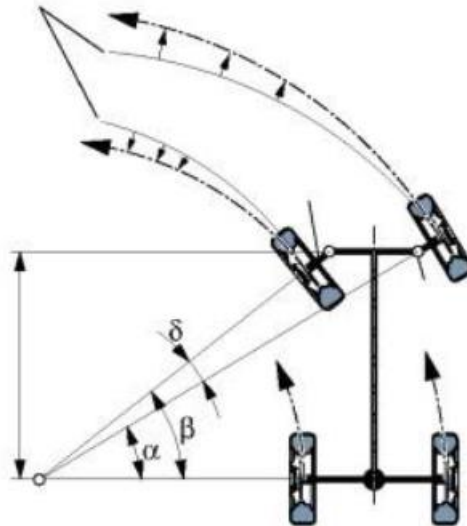


Figure 6.2. Ackermann geometry.[28]

The instantaneous centre of rotation is the point around which a body rotates at a given instant. It is defined as the intersection of the perpendiculars to the trajectories of the points of the body in motion around which a body rotates at a given instant. It is defined as the intersection of the perpendiculars of the trajectories of the points of the body in motion. [29]

According to Ackermann, the angle formed by the outer wheel with the rear axle extension (α) must be smaller than the angle formed by the inner wheel (β).

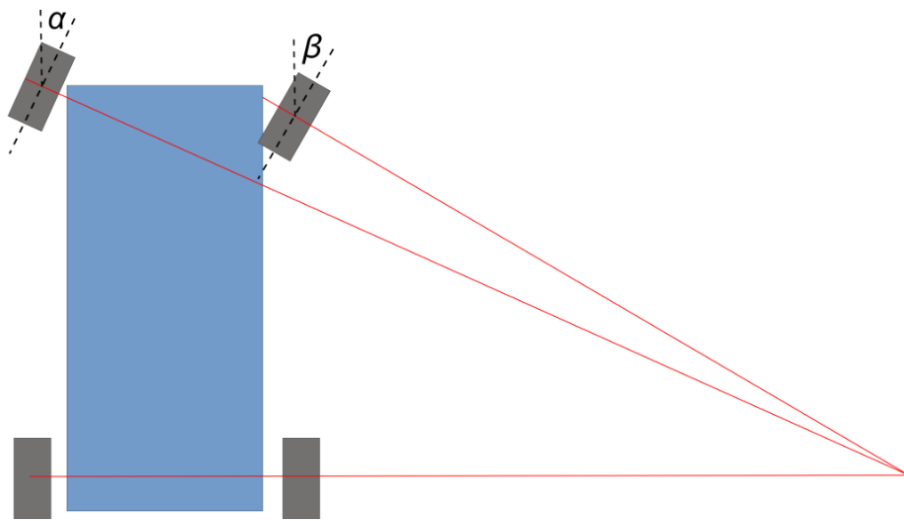


Figure 6.3. Ackermann's principle.[30]

If a vehicle is designed without regard to Ackermann's principle and the two front wheels rotate at the same angle, they are not rotating relative to the same point.

If Ackermann's principle is applied, the inner wheel will rotate at a greater angle so that the instantaneous centre of rotation is the same for all wheels. To achieve this effect, the steering rods shall form a certain angle to the longitudinal axis, as described in the figure 6.4.

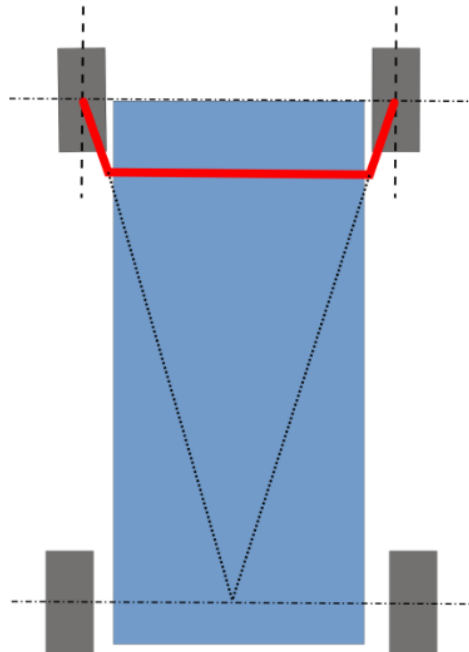


Figure 6.4. Steering rods.[30]

The following formula is useful to determine Ackermann: [31]

$$\frac{\text{track}}{\text{wheelbase}} = \cot \alpha - \cot \beta$$

$$\text{Ackerman} = \cot^{-1}(\text{wheelbase}/(\text{wheelbase}/\tan \beta) - \text{track})$$

$$\text{Percentage} = 100 * \left(\frac{\alpha}{\text{Ackerman}}\right)$$

Graphically we can identify that with 100% Ackermann the connecting rod extensions are cut off in the centre of the rear axle. If the percentage is higher than 100, they will be cut in front of the rear axle; and at the rear if it is lower. Basically, the Ackerman percentage indicates how much the inner wheel rotates compared to the outer wheel. With 100% Ackerman the turning circles of the inner and outer tyres are concentric, i.e. they share the same instantaneous centre of rotation; while 0% means that both circles are identical, i.e. the inner wheel turns at the same angle as the outer wheel.

In practice tyres are deformed. This deformation is known as the drift angle and is the difference between the angle of rotation and the angle that the tyre contact patch actually acquires due to the forces exerted on it, as defined in previous chapters.

To achieve the desired effects in geometry, the concept of static convergence must be introduced.

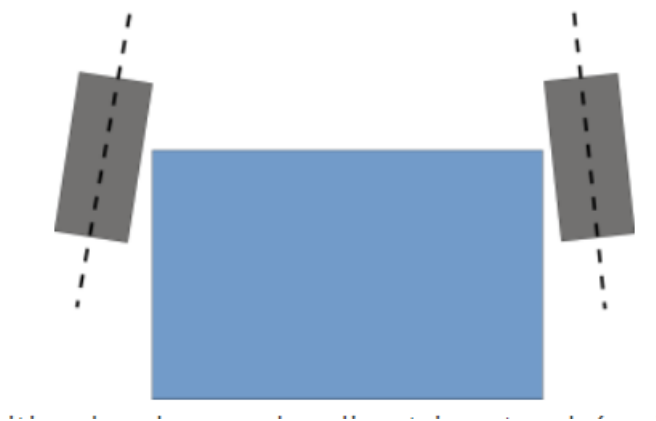


Figure 6.5. Static convergence.[30]

If the toe-in is positive, the two steered wheels will have some convergence towards the centre of the paths, i.e. the inner wheel will try to describe a slightly larger circumference and the outer wheel a slightly smaller circumference than the

one it is following. With this geometry, the drift angle of the inner wheel is reduced and that of the outer wheel is increased.

In case the convergence is negative, the inner wheel will try to describe a smaller circumference than the one it is following and the outer wheel a larger circumference. With this geometry the drift angle of the inner wheel increases and that of the outer wheel decreases. [12]

5.3. Steering gears

The steering rack or steering box translates rotary motion of the steering wheel to linear motion at the tie rods. In turn, the tie rods translate this linear motion back to rotary motion about the kingpin axis resulting in steer of the front wheels. [3]

It is known that there are many connections in the steering system, which can have a certain amount of play which makes the steering inaccurate, as the driver will not know exactly in which direction the wheels are pointing when turning. Steering system components must be mounted firmly and securely to ensure safety and control.

The overall steering ratio is defined as degrees of steering wheel angle divided by corresponding front wheel angle. For race cars it varies from over 20:1 for Superspeedway cars to less than 10:1 for Formula One cars on tight street circuits. Higher (20:1) ratios will require less effort than lower (quicker) ratios.

Rack and pinion gearsets convert rotary motion at the steering wheel to linear motion at the inner tie rod ball joint. The steering ratio is calculated using the rack c-factor and the steering arm length (as measured from the outer ball joint to the kingpin axis). [3]

$$c - factor = \frac{travel}{360^{\circ} pinion\ rotation}$$

Once the c-factor is known for the rack, the steering ratio can be calculated approximately by

$$Steer\ ratio = \sin^{-1}\left(\frac{c - factor}{steering\ arm\ length}\right)/360$$

Reverse efficiency is the name given to the ability of the steering system to transmit road irregularities to the driver. Rack and pinion steering has a high efficiency, most of the torque on the kingpin will appear at the steering wheel. This is beneficial for tracks with a clean surface where these aligning torque signals will help the driver determine the limit of front tyre grip.

Steering dampers are a solution for roads with many imperfections. The steering damper is designed to offer low resistance to movement at low speeds but much higher resistance at higher speeds. Steering dampers reduce the driver's sensation if there are many bumps at low speeds. This makes handling at the limit more complex, for example in a slalom. For this reason, their use is not recommended.

Power steering is used on many production vehicles used for racing, such as NASCAR, Trans-Am and GTO. The reason they are used is to reduce the amount of effort required from the driver. The problem with power steering is that the units are usually modified production parts which have a built-in nonlinear characteristics. The force gain is approximately linear around center, as controlled by a spring, and once the spring is deflected significantly the gain increases dramatically. This non-linearity is not a good thing for the driver, as the sensations received from the front tyres are very important. [3]

Another way to reduce the driver's steering effort is to design a steering system with a very small kingpin and caster angle, a minimum scrub radius and a small amount of trail.

5.4. Bump and roll steer

Generally, steer that results from wheel ride (bump) motion and body roll motion is undesirable. The reason for wanting a zero bump steer is very simple, if the wheel steers when going over a bump or when the car rolls in a turn, then the vehicle will trace a different trajectory to the one selected by the driver. [3]

Bump steer is when the front wheels move up and down, we want the front wheels to maintain a particular direction. It's most important for the wheels to have

minimal bump when negotiating turns. There are certain elements of the construction of the front end components that will make this happen. [32]

The angles of the upper and lower control arms, meaning a line extending through the center of rotation of the ball joints and inner mounts of each arm, intersect at a point we call the instant center (IC). This is one of the components used to determine the moment center location. In order to have near zero bumpsteer, the intended goal, we need to have the tie rods on each side point toward the IC for its side. This is one of two criteria for near zero bumpsteer. The other thing we need is for the tie rod to be a specific length. That length must be equal to the distance formed by 1) a line extending through the centers of rotation of the tie-rod ends, and 2) the tie-rod line intersection with a) lines extending through both the upper and lower ball joints, and b) the plane that passes through the inner chassis mounts. This can get a little complicated because although the ball joints do form a single line, the chassis mounts form a plane because of the front and rear mounts. So, the inner tie-rod intersection point is where the tie-rod line intersects the plane of the inner mounts and the outer line intersection point is where it intersects the ball joint line.

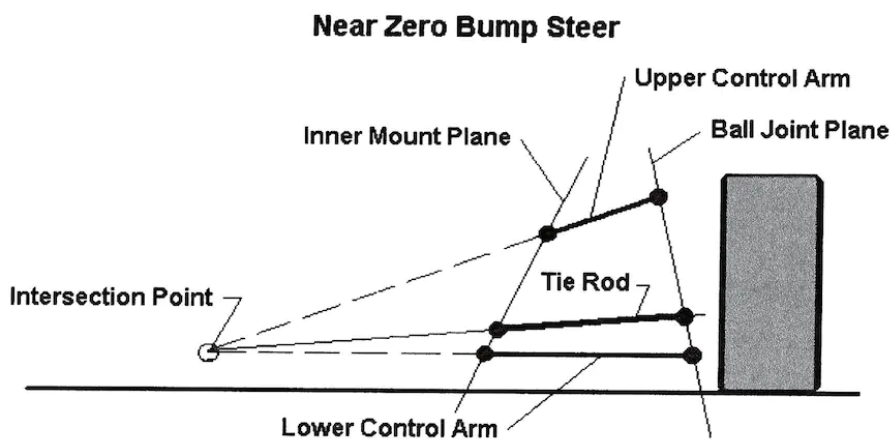


Figure 6.6. Near zero Bump steer.[33]

Let's now analyse what cause bumpsteer. As the wheel moves vertically, the wheel will either steer left or right. If the tie rod was pointed so the tie-rod line passes below the IC, then the wheel will bump-in (toward the centerline of the car) as the wheel travels up, and bump-out when the wheel travels down. If the tie-rod line passes over the IC, then we will have bump-out as the wheel travels up, and bump-in when the wheel travels down.

If the tie rod were too short, we would have bumpsteer in when the wheel travels in both directions from the static ride height position. If it were too long, then the wheel would bump-out as the wheel traveled in both directions from ride height. [34]

However, we can use deliberate amounts of bump steer to alter the behaviour of the vehicle in cornering. With a very small amount of toe-out in bump will decrease the slip angle of the outside front tire at small steering angles during the corner entry phase. This often reduce corner entry understeer. If we put too much, the vehicle will become dynamically unstable over bumps and under the brakes.[33]

The simplest method to measure bump steer is swinging the wheels individually through their range of vertical travel and measuring the change of toe-in that occurs. Spring and shock absorbers units should be removed from the vehicle. Place the car on ride height blocks and lock the steering rack in its center position. Then mount a flat aluminium plate rigidly to the hub flange or brake disk. The plate has horizontal lines scribed on its surface to indicate one, two and three inches of bump and drop travel. [35]

Bump steer can be beneficial in some cases. For example, causing the rear axle wheels to turn when travelling vertically upwards will provide "active toe-in" in cornering. This can be used on a front-wheel drive car to generate divergence at the rear axle during cornering to encourage oversteer. [36]

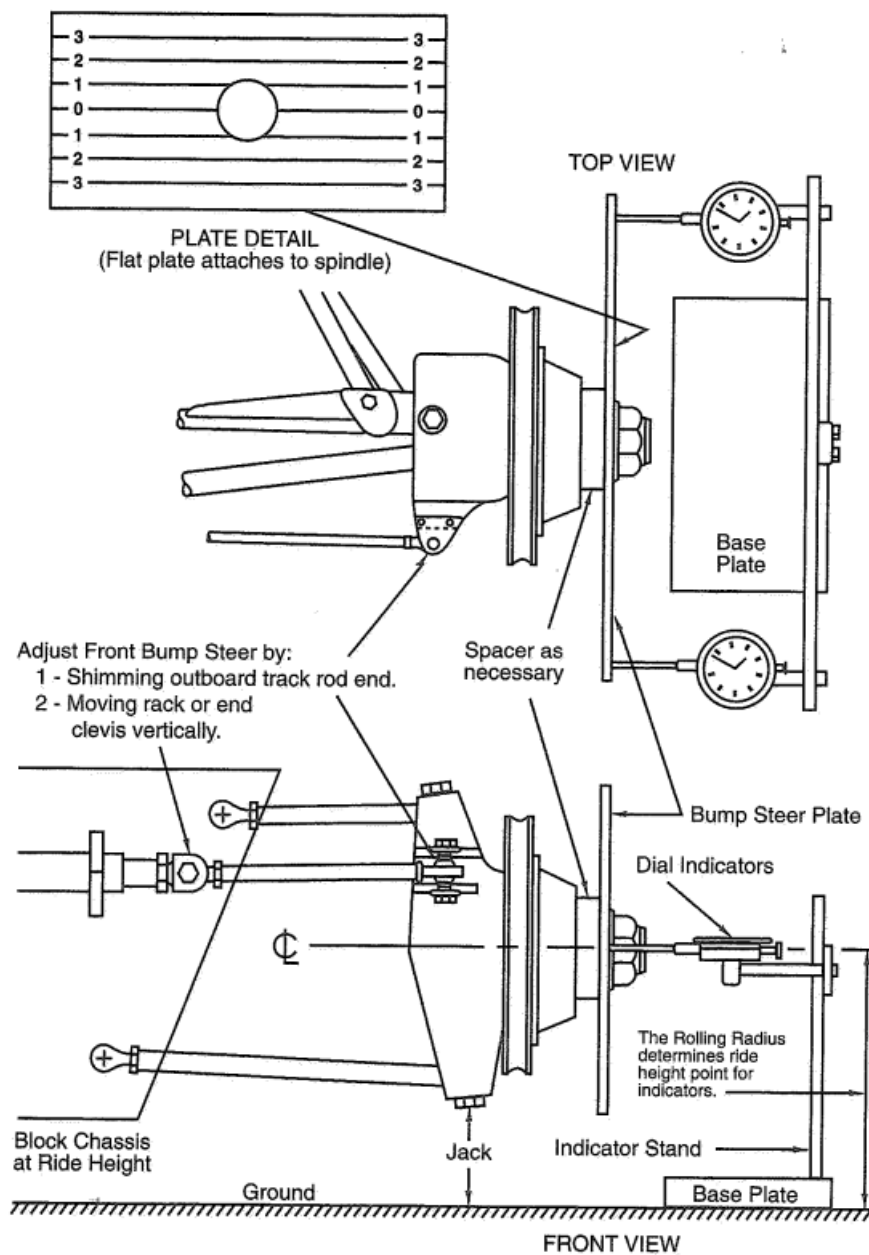


Figure 6.7. Measuring bump steer.[35]

Roll steer is a pretty complex phenomenon. It is basically the self steering action of any automobile in response to lateral acceleration and consists of slip angle changes due to camber change, to change and the inertias of the unsprung mass. There is no much to do to solve this problem other than reduce gross weight, CG height and polar moment of inertia.

Figure 6.8 shows vehicle cornering force vs average tire slip angle. The various aspects of self steering, bump steer, roll steer and deflection steer affect the slope

of the lower part of the curve, this is to say, the transient period when we are building cornering force. [37]

In the normal racing corner sequence, the car is either accelerating or decelerating almost all the time and so is in a constant transient state with regard to load transfer and slip angle. Transients are all important to total performance, good transient response makes the car more pleasant to drive. [38]

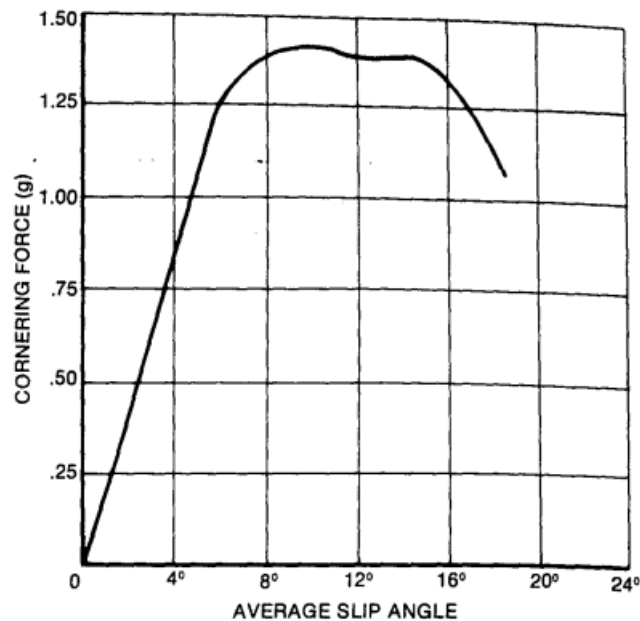


Figure 6.8. Cornering force v average slip angle.[33]

5.5. Toe in and stability

Toe in between a pair of wheels, at either end of the vehicle is a dynamically stable condition. If load is transferred laterally between a pair of wheels, by a bump or a wind gust, for example, the load transfer will cause a relative increase in the slip angle of the more heavily laden wheel. If the wheels should be toed out when this occurs, then the deflection will cause the vehicle to steer towards the inside wheel. This can be most upsetting at the front of the car. At the rear, it will make the car undriveable. On the other hand, if the wheels are toed in, the vehicle still steers toward the inside wheel, but that wheel is pointed in the direction that we want the car to go and the vehicle is dynamically stable.

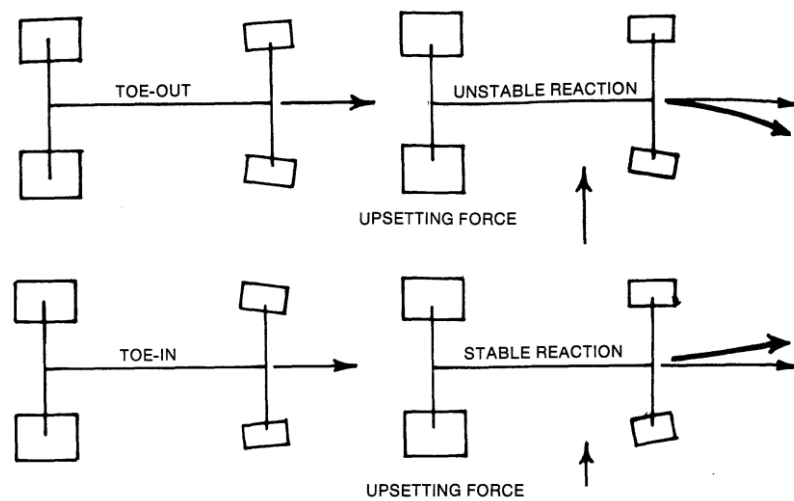


Figure 6.9. Effects of toe in and toe out on directional stability.[33]

Toe in is when the plane of the wheel is inclined forwards with respect to the longitudinal axis of the car, i.e. when viewed from the front, the wheels are closed forwards. Toe out is when the wheels are tilted in the opposite direction, i.e. when the wheels, when viewed from the front, are open. [39]

When the car is in motion, the suspensions compress or extend due to ground irregularities and weight transfer, therefore, the toe in or toe out values may change. On the front axle, when braking, the arc of circumference described by the steering arms may be different from that of the suspension arms. Depending on how these arcs intersect, the wheels will either toe in or toe out. These steering changes, which steer the wheel at angles undesired by the driver, are, in principle, counterproductive. If the wheels diverge under braking, this manoeuvre will be unstable and it will be difficult to insert the car into the corner.

When the car is set up for track racing and is rear-wheel drive, it is not acceptable for the outer wheel of the rear axle to develop divergence. Under these conditions the car will be oversteering and uncontrollable, especially in fast corners. Some manufacturers induce a minimum toe-in value on the outside wheel when cornering as the suspension compresses. In this case, the inside wheel will develop divergence, but both will be pointing into the corner, favouring a certain understeer tendency. [12]

However, when the corner is unfavourable or if the toe-in increase is too great, the rear will be so far inboard that the driver will have to correct the steering as soon

as the car is pointed towards the corner apex. In an understeering car, in the first phase of cornering it is desirable for there to be some divergence of the loaded wheel at the start of the corner and for this to disappear as the roll is completed.

To summarise, it can be deduced that for cornering it is convenient to have divergence at the front wheels. The inside wheel will have a tighter turning radius than the outside wheel. Therefore, ideally, the inside tyre should be steeper into the corner than the outside tyre to maximise grip and stability (Figure 6.10). In the case of toe in, if one wheel is at the correct turning angle, the opposite wheel will be further away from its ideal turning angle, causing drag and reluctance to corner. In general, race car vehicles have toe out and a negative camber at the front wheels and toe in in the rear wheels to balance the effects.

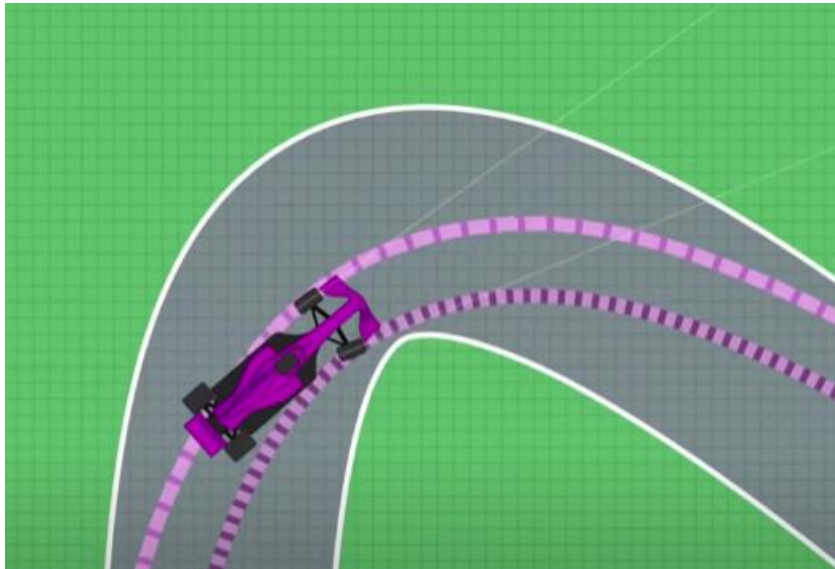


Figure 6.10. Turning with toe out.[40]

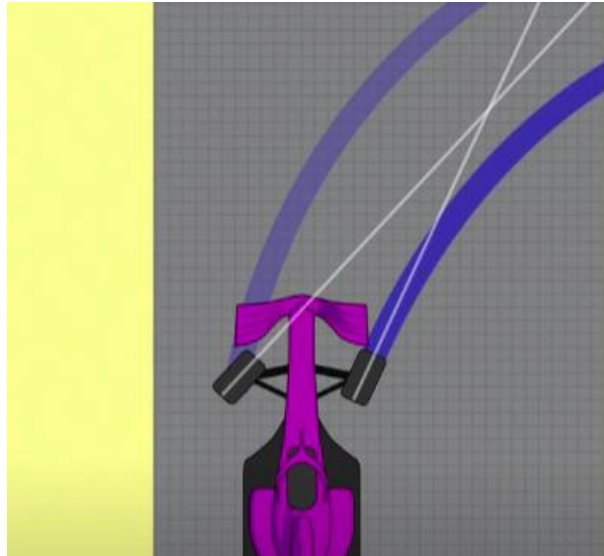


Figure 6.11. Turning with toe in.[40]

With regard to straight line stability, toe out will make the car unstable in the event of any irregularities in the road surface or forces outside the car, as it is more sensitive to cornering. Conversely, a vehicle with a toe in, because it is more reluctant to turn in at the start, is more stable in a straight line. One of the latest advances in this matter to date is the DAS (Dual Axis Steering) system developed by the Mercedes-AMG Formula 1 Team, and consists of a mechanism that allows drivers to vary convergence or divergence depending on the situation. For example, they can apply toe when entering a corner or they can put the wheels parallel (no toe-in or toe-out) in straight lines to increase top speed and minimize tire wear.

7. Handling

The word handling is often used in the automotive industry to describe how the vehicle behaves when turning. A car with "better handling" generally is faster when cornering and is less likely to lose control suddenly. Tyres, steering and vehicle weight all play a role. Although for a more detailed study it would be necessary to take into account also the effects of aerodynamics and different suspension configurations. It is often thought that handling is only related to racing vehicles, but it is very important to have a car with good handling also for emergency situations, as the vehicle will react in a predictable way to the driver's steering and braking inputs.

Depending on the characteristics of the study vehicle, we will need a trade-off between suspension and handling. Normally, better handling requires stiffer suspension, which makes the ride less comfortable, as road imperfections will be more noticeable and the car will "bounce" a lot.

As mentioned above, the weight of the vehicle is also important in handling due to inertia. A very heavy vehicle will tend to go straight when the steering wheel is turned into a corner. Also the centre of gravity is important, a car with too high a centre of gravity will tend to make the vehicle lean too much, causing a sensation of tipping.

7.1. The G-G diagram.

The g-g diagram or force diagram is one of the most important graphs available when analysing the handling and set up of cars. It is a representation of the forces to which the vehicle is subjected at any given moment. Throughout each turn, this instantaneous representation of the forces acting on the vehicle will complete a kind of picture as shown in Figure 7.1.

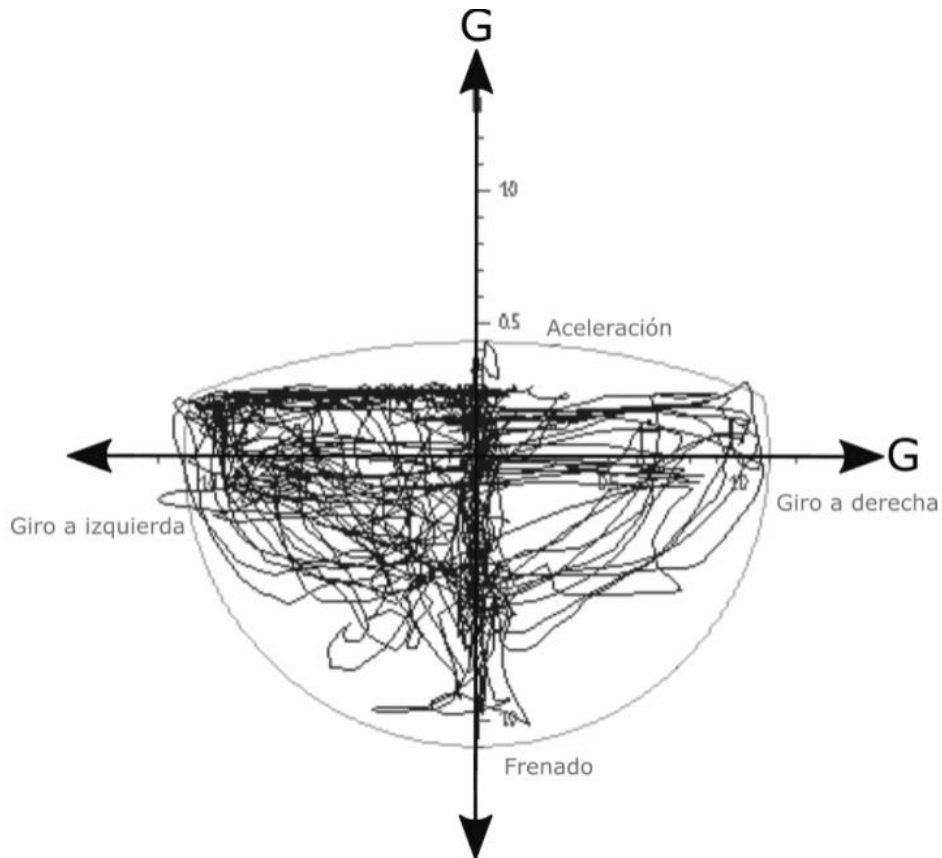


Figure 7.1. G-G diagram.[41]

Basically, the function of this diagram is to check whether the driver or the car is making full use of the tyre's capacity. The contact patch of a tyre is generally oval-shaped, withstanding more lateral forces than longitudinal forces. To make full use of the tyre's capacity, the driver should move around the contact patch. A serious mistake by a driver can be to brake, suddenly release the brake pedal and take the corner, and so on through all the corners of the circuit. A graph showing that the driver is not riding correctly and is not using the full capacity of the tyre is shown in figure 7.2.

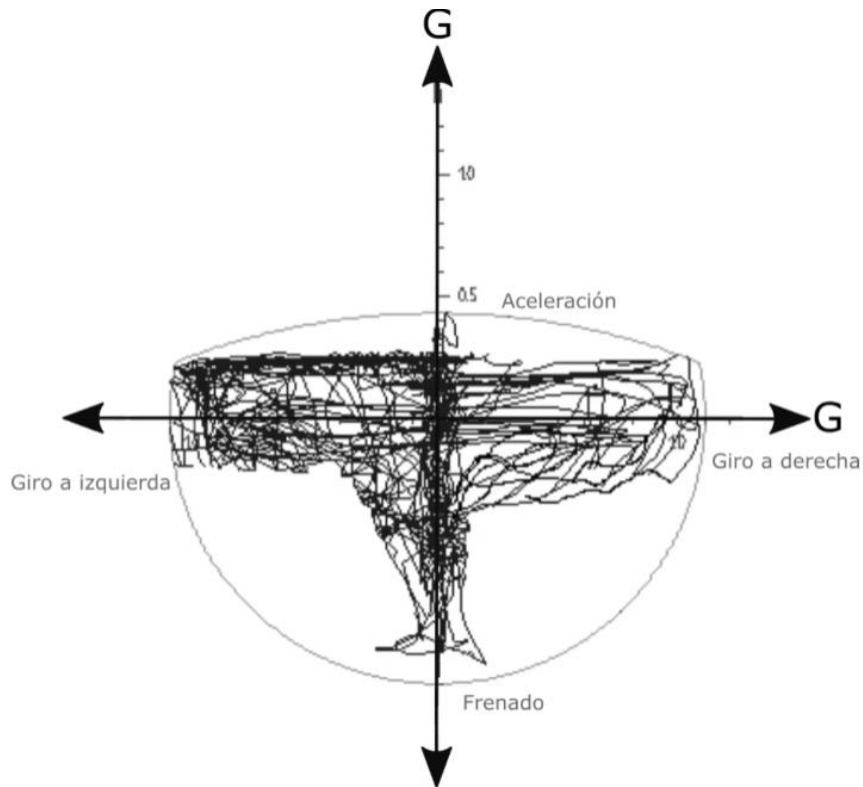


Figure 7.2. Driver not using the tyre to its full potential.[41]

To take advantage of the tyre's capacity, the driver should brake hard and as he releases the brake pedal, he should turn slightly into the corner. In other words, the aim is to be progressive, both at the entry and exit of the corner, in order to complete the whole g-g diagram, as in figure 7.3.

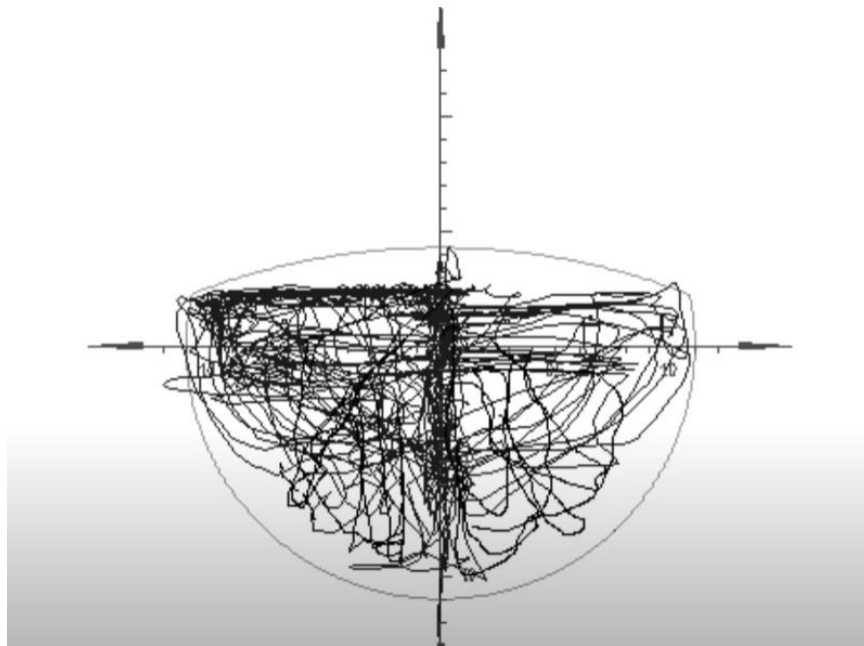


Figure 7.3. Nice driving.[41]

Abel Caro has developed a new version of the G-G diagram, because in this diagram it is very difficult to know in which part of the wheel is losing time, as this diagram is in the form of lines. In this new system, the diagram is not in the form of lines, but in the form of dots. When a wheel is detected to be sliding, it is drawn with a triangle, or when a wheel is blocking, it is painted with an x. Moreover, this diagram is separated both by corners, generating different graphs for each one of the corners, and by speed ranges, knowing the aerodynamic load for each one of them, and also being able to know if the driver is performing better in fast or slow corners.

7.2. Car behaviour

It has already been mentioned that any lateral force acting on the centre of gravity results in a deformation of the tyres, causing slip angles. This is the angle between the direction of the rim plane and the actual direction of the tread in contact with the asphalt. In a side wind or corner, there will be slip angles on the front and rear tyres.

7.2.1. Variables in action

The final value of the slip angle depends on the interaction of all suspension variables, spring stiffness, anti-roll bars, inflation pressure, etc.

When the front slip angles are higher than the rear slip angles, the car is understeering, with the front wheels pointing towards the centre of the turn and the nose of the vehicle pointing outwards, as if it is trying to go straight. The car turns less than the driver commands with the steering wheel. On the other hand, when the rear slip angles are larger than the front, the car is oversteering, turning more than the driver commands with the steering wheel and the rear of the car wants to overtake the nose, going too far into the corner and crossing the car.

7.2.2. True radius of the corners

If slip angles did not exist, the car would rotate around an instantaneous centre of rotation located at the intersection of the perpendiculars to the plane of its wheels,

as shown in figure 7.4. But what happens in reality is that the car has slip angles, which makes it tend to rotate around an instantaneous centre of rotation defined by the intersection of the vectors representing the cornering forces of the front and rear tyres.

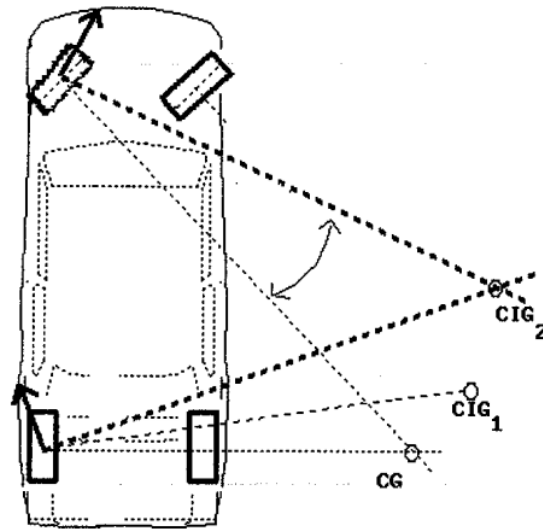


Figure 7.4. Instantaneous centre of rotation if there were no slip angles.[39]

When the car has higher front slip angles than the rear ones, the crossing of the tyre vectors moves forward, in CIG₂, following a larger corner radius.

If the front slip angle continues to increase, the radius of the corner will become larger and larger and the car will turn less and less, until it runs off the outside of the corner. The driver can resolve this situation before going outside the track limits. One might think that a solution might be to turn the steering wheel more, however, this is a mistake. If the front tyres have already exceeded their grip limit, increasing the slip angle will reduce grip even further. The driver can also brake, but again, this is counterproductive. With the tyre completely crossed over from the true direction of travel, there is a resistance to forward motion but also a tendency to skid, and by applying hard brake pressure, the tyre will lock up and the car will continue out of the corner.

The driver can take the throttle off and open the steering slightly. Opening the steering decreases the drag, and by taking the throttle off, speed is lost, so the car's centre of rotation is reduced and the car shifts backwards a little. The

decrease in speed lowers the slip angle, allowing the tyre to regain its coefficient of friction and grip.

7.2.3. Rally driving

There are few situations where it is beneficial to apply the brakes with an understeering car that has exceeded its limits. In slow corners or when you start to corner and you have a lot of braking on the rear axle, and you see that the car is starting to nose out. A short pressure on the brakes will cause the rear end to cross over, making the car oversteer, and pulling the car into the corner. This is a technique often used in rallying. In such situations on dirt roads, it is advisable to have excess brake on the rear axle and to align the rear tyres with a certain toe out.

This way, when braking, the rear of the car tends to cross over. It is also possible to brake with the left foot while accelerating, to balance the understeer tendency in the middle of corners, allowing the characteristics of all-wheel drive and front-wheel drive cars to be better exploited.

7.2.4. Crossed car

Figure 7.5 shows what happens when the rear tyre slip angles are larger than the front tyres.

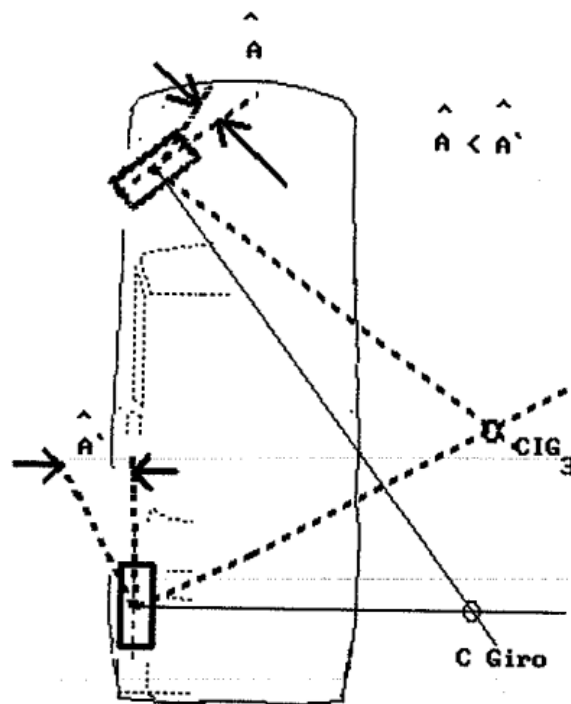


Figure 7.5. Oversteering car.[39]

You can see that the vector of the rear tyres points further forward, so the actual centre of rotation (CIG3) is moved further forward and the radius of the corner is reduced. If the speed at which the corner is taken is maintained, the centrifugal force increases due to the decrease in radius. In this situation, because the grip of the rear tyres has reached the limit, they will start to skid and a spin may occur. It is an unstable condition, and more unsafe than the understeer condition. The driver may be able to overcome this but must not brake or he will lock up the wheels and spin out. One of the things the driver can do is to slow down by lifting the accelerator pedal if you are going at low speed and the loss of grip has been slight and he can also at the same time turn the steering wheel away from the corner (countersteering), increasing the radius of the corner and decreasing the centrifugal force.

This double manoeuvre would serve to regain control of the vehicle, however, during this time the vehicle has not been accelerated, so time would be lost. Alternatively, the driver can accelerate by countersteering. Although the tractive effort will decrease, the cornering force exerted by the tyre, the forward thrust and the fact that the car is completely crossed with respect to the real trajectory it is following, allows the driver to continue to counteract the tendency to spin.

7.2.5. Front engine - front-wheel drive

A vehicle with 65/70% of its weight on the front axle and with a relatively low roll centre will be an understeering car on corner entry and exit, which will be even more accentuated if the vehicle is front-wheel drive.

A rear-wheel drive car with the same characteristics and a lot of power could momentarily become unbalanced under hard acceleration and even oversteer, which can be observed when accelerating on wet road surfaces.

The car could also be balanced on corner entry, or even in the middle phase of corners, by stiffening the rear springs or the rear anti-roll bar.

In racing, a car with a quick response to steering changes is ideal. However, a stiffer car is more nervous, and more difficult to drive as it requires more precision and finesse in handling.

7.2.6. Rear engine

This type of car is common in single-seater and prototype categories, which are very well adapted to the needs of a fast, dry track with good grip. When braking, weight is transferred to the front end, increasing the front end's grip.

When turning the steering wheel, the weight is transferred to the outside front wheel. During this first phase of the corner, the car should ideally respond immediately to the driver's input. It is good to have a slight understeer, so that the driver, anticipating it, can compensate for the slip angle of the tyres and make the car follow the desired line.

In the middle phase of the corner, some power will be applied to maintain transit speed or the throttle will have been taken off completely. Again, the car would be slightly understeering, or even neutral. At the exit of the corner, when the driver starts to accelerate, it is no longer desirable for the car to be understeering, but a neutral or slightly oversteering attitude is desirable when applying full torque.

If the car is understeering at this stage of the corner, the weight transfer to the rear under acceleration would cause even more understeer, which would not only

increase the induced tyre drag, but could also lead to a loss of acceleration and tyre spin. In addition, the car would still be pointing out of the corner, so the driver would be forced to take some or all of the throttle off, losing time. This is why a slightly oversteering car is needed for corner exit, allowing the driver to accelerate earlier and more. A well dosed throttle and a certain degree of counter-steering will allow the driver to align the nose to the exit earlier and accelerate sooner.

In low speed corners there will be a certain amount of excess torque which will eventually induce some oversteer. A good driver can use the throttle as if it were another steering wheel, and will know how to control it so as not to make the tyres slide too much.

So far it has been argued that a rear-engined car will be beneficial in corner entry and mid-phase. While the transfer of weight to the rear just under acceleration benefits rear axle grip, the same transfer takes weight off the front axle when the car is still turning, which would lead to a tendency to understeer. This excessive understeer at the exit of slow corners is one of the most difficult problems to solve.

To deal with it, you must first eliminate understeer, if any, at the corner entry. It must also be ensured, by means of shock absorbers or anti-roll bars, that the attitude changes to oversteer when accelerating.

7.2.7. Front wheel drive vehicles

This is most often the case for series production cars in touring car racing and some rallies. Front-wheel drive has prevailed over rear-wheel drive because of the ease of assembly at the factory and because these cars offer greater roominess. Another reason, dynamic, is that these cars are inherently understeering, so they are easier to control in complex situations.

Front-wheel drive can outperform rear-wheel drive in wet, muddy or uneven road conditions.

A major problem they face is braking. In these cars, a higher percentage of braking must be applied to the front brakes, wasting some of the braking potential of the rear brakes. Another problem is during acceleration, when the accelerator pedal is applied, weight is transferred to the rear, but the drive axle is deprived of

weight when it is most needed. A similar thing happens on corner exit, as the almost weightless inside front wheel is the first to slide. These cars require a special driving technique, forcing the driver to steer differently and in some cases to use the brake with the left foot while continuing to accelerate until the rear wheel loses grip and the car, becoming oversteering, pushes the nose into the corner.

The slower the track and the tighter the corners, the more the problem of understeer is accentuated.

7.2.8. Set up for an understeering car

Work must be done to prevent the front tyres from getting too hot or too worn, as on these cars, the front tyres work much harder than on a rear-wheel drive car, in addition to cornering, they carry a higher proportion of weight, braking, traction and some slippage. The work done by the tyres is lightened by a relatively soft suspension at the front. This also contributes to less weight transfer between the two front wheels, increasing axle grip. When trying to balance the tendency of the car to be oversteer or neutral, the rear suspension stiffness is made much stiffer.

As wider, racing tyres are used, it is possible to use stiffer springs at the front. Soft springs force the maximum possible negative camber value to be used, so that the front roll will cause the tread temperatures to be even across the full width of the tyre.

In this type of car, the front axle can be toe out. Toe out at the front reduces understeer on corner entry and makes the inside wheel work better. Generally on rear-engined, rear-wheel drive cars, toe out on the front axle is unacceptable for circuit racing and very fast tracks. However, in rallying and slow dirt tracks, because of the need for the nose to register in the corners, front-engined, front-wheel drive cars can accept a little toe out on the rear axle. As long as straight line stability does not suffer too much and the car is progressively controllable, toe out at the rear is acceptable.

7.2.9. Driving techniques

As mentioned above, front-wheel drive rally cars are comfortable on dirt roads and in low grip conditions. In these and in these circumstances, a technique called "Nordic" can be applied, which was introduced to the rally world by Scandinavians who are used to driving on ice.

To begin with, you need the rear brakes to lock up quite easily, or there needs to be some toe out on the rear axle, to make it easier for the car to oversteer.

The technique is to make the rear of the car slide during braking towards the same side as the side to be turned. Once this is achieved, the driver quickly turns the steering wheel to the side of the corner. The car then reacts and tails to the opposite side. Once the nose of the car is in line with the corner, the driver proceeds to control it with the throttle. Similarly, if the corner is relatively long and the nose wants to come out again, the driver, while continuing to accelerate, gently squeezes the brakes and removes grip from the rear tyres, thus balancing the car and allowing it to continue accelerating. With an all-wheel drive car it is possible to take this technique to very high limits as the rear axle is more controllable. If you try to do the same with a rear-wheel drive car, the car may cross over too much, and you will lose more time, especially in a succession of corners.

7.3. Handling characteristics, causes and effects.

7.3.1. Instability

Straight line instability – general

Possible causes

- Rear wheel toe out, either static due to incorrect setting or dynamic due to bump steer.
- Vast lack of rear downforce or overwhelming amount of front downforce.
- Broken chassis or suspension member or mounting point.
- Great amount of front toe-in or toe-out.

Straight line instability under hard acceleration

- Limited slip differential worn out.

- Insufficient rear wheel toe-in

Straight line instability – car darts over bumps

- Too much front toe-in or toe-out.
- Uneven front castor setting
- Uneven front shock forces or bump rubbers
- Front anti-roll bar too stiff

Instability under the brakes – front end darts

- Too much front brake bias

Instability under the brakes – car wants to spin

- Too much rear brake bias or too much positive camber on rear tires.

7.3.2. Response

Car feels generally heavy and unresponsive

- Too much aerodynamic downforce

Car is slow to take a set in corners, rolls a lot

- Too little shock absorber damping.
- Insufficient roll resistance

Car responds too quickly – slides at slightest provocation

- Too little downforce
- Too stiff in either ride or roll resistance
- Too much shock
- Too much tire pressure

7.3.3. Understeer

Corner entry understeer – won't point in

- Insufficient track width

- Front roll stiffness too high
- Front roll center too low
- Insufficient front shock absorber bump resistance
- Insufficient front downforce
- Excessive dynamic positive camber on outside front tire
- Braking too hard and too late

Corner entry understeer – car initially points in and then fades away

- Too much front toe-in
- Insufficient front downforce
- Insufficient front roll camber compensation
- Non linear load transfer due to roll axis inclination
- Insufficient front wheel travel in droop.
- Too little front shock bump resistance

Corner entry understeer – car points in and then darts

- Insufficient front wheel travel in either bump or rebound
- Nose being sucked down due to ground effect

Corner exit understeer – slow corners

- We must reduce the front shock rebound forces and improve throttle response.

7.3.4.Oversteer

Corner entry oversteer

- Lack of rear downforce
- Broken outside rear shock or front anti roll bar
- Limited rear suspension travel

Corner exit oversteer – gets progressively worse from the time that torque is applied

- Worn out limited slip
- Too much rear roll stiffness

- Too much rear camber
- Too little downforce
- Too little rear toe-in

7.3.5. Ride and roll rates

Cause listed first

Too much spring – overall

Car will not put power down on corner exit, excessive wheelspin. Much unprovoked sliding.

Too much spring – front

Initial understeer, although car may point into corners well. Front end breaks loose over bumps in corners. Front tires lock over bumps.

Too much spring – rear

Oversteer immediately upon power application coming out of corners. Excessive wheelspin.

Too little spring – overall

Car contacts race track a lot. Floating ride. Car is slow to take its set

Too little spring – front

Excessive roll on corner entry. Initial understeer, car won't point in

Too little spring – rear

Excessive acceleration squat and accompanying rear negative camber.

Too much anti roll bar – overall

Car will be very sudden in turning response and will tend to slide

Too much anti roll bar – front

Initial corner entry understeer which usually becomes progressively worse as the driver tries to tighten the corner radius

Too much anti roll bar – rear

Corner exit oversteer, excessive sliding coming out of corners

7.3.6. Shock absorber forces

Too much shock – overall

Very sudden car, much sliding. Car does not absorb road surface irregularities.

Too much rebound adjustment

Wheels do not return quickly to road surface after displacement

Too much bump adjustment

Initial bump reaction very harsh

Too little shock – overall

Car floats a lot in ride.

Too little rebound adjustment

Oscillates after bumps and does not put power down well

Too little bump adjustment

Car dives or squats a lot.

7.3.7. Wheel alignment

Front toe in – too much

Car darts over bumps, under the brakes and during corner entry. Car won't point into corners.

Front toe out – too much

The car swerves under the brakes and can be unstable in a straight line especially in wind gusts for example

Rear toe in – too much

Rear feels light and unstable on corner entry

Rear toe in – too little

Power on oversteer, during corner exit

Rear toe out – any

Straight line instability

Front wheel castor – too much

Excessive physical steering effort and too much self return action.

Front wheel castor – too little

Car too sensitive to steering. Too little steering feel, self return and feedback

Front wheel castor, uneven

Steering effort harder in one direction than in the other

Car swerves in one direction (toward the side with the high castor setting) in a straight line

Camber, too much negative

Inside of tire excessively hot or wearing too quickly. At the front, reduced braking capability and at the rear, reduced acceleration capability.

Camber, too much positive

Outside of the tire will be hot and wearing. This should never be and is almost always caused at the rear. Will cause corner exit oversteer and reduced tractive capacity and even corner entry instability. At the front it is usually caused by excessive chassis roll or by insufficient roll camber compensation in the suspension linkage and will cause understeer after the car has pointed into the corner

7.3.8. Tires

Too much tire pressure

Excessive sliding and wheelspin. High temperature at center of tire.

Too little tire pressure

Soft response and high tire temperatures.

8. Conclusions

Before starting this project, my initial goal was to try to explain in a clear way how the different components of a vehicle affect its performance and manoeuvrability. I considered that handling is one of the most complex topics in the automotive field, as it depends on many factors, including the human factor, which can make it unpredictable sometimes.

I believe that with this paper the initial objectives have been satisfactorily fulfilled, despite the fact that important elements such as suspension and aerodynamics have not been studied. These sections were outside the scope of the project, as if they had been included, it would have been too extensive.

A study of the behaviour of tyres and the forces acting on them under different circumstances has been carried out. A study on the tyres should always be fundamental in projects of this nature, as it is the only part of the vehicle that is in contact with the ground. In addition, the dynamics and reactions of the vehicle as well as its stability in curves and straight lines have been analysed, thanks to the chapters on braking and steering.

Chapter seven could be a summary of handling, taking into account the human factor and how the driver feels the car's behaviour, and the theoretical design objectives that should be taken into account when designing a high-performance vehicle.

Although this is a subject that has been studied for many years, I have been able to appreciate that there is still some room for improvement, especially in the interpretation of the driver-car communication and in the changes in the set-up depending on the driver's feedback. This is not a written rule, because of the human factor, the driver may feel better with an understeering car than with an oversteering car, or vice versa. The role of the engineer is to understand the driver, and try to give him/her the best car by knowing the driver's preferences and driving style.

Personally, this project has made me acquire fundamental knowledge of great importance for what I want my profession to be, thanks to the careful reading of several papers, books and thesis of other students.

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