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Development of crown profile models for *Pinus pinaster* Ait. and *Pinus sylvestris* L. in northwestern Spain

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We used data from *Pinus pinaster* Ait. and *Pinus sylvestris* L. trees growing in single-species even-aged stands in northwestern Spain to develop crown profile models. Such models are key components of growth and yield models, and they are also important for assessing competitive level, microclimate, tree vigour, mechanical stability, biological diversity, fire susceptibility and behaviour under wind stress, amongst other features. The equations used in crown profile estimation (i.e. those including crown radius, largest crown radius, height to the largest crown radius and height to the base of the live foliage) were fitted simultaneously to take into account correlations between the different variables. The fitting technique also enabled us to consider an autocorrelated, heteroscedastic error structure and to use a different number of observations for the different variables involved in the fitting process. The fitted models explained between 66 and 94 per cent of the variability in crown radius, with a mean error between 0.15 and 0.64 m. The crown profile models developed can be used to estimate the crown profile when only diameter at breast height, total tree height and height to the base of the live foliage are measured. Inclusion of more variables (such as largest crown radius) provides more accurate results.

Introduction

Maritime pine (*Pinus pinaster* Ait.) and Scots pine (*Pinus sylvestris* L.) are the first and second conifer species in terms of area covered in Galicia (NW Spain). They occupy ~383 000 and 63 000 ha, respectively, as dominant species, which is equivalent to 27.4 and 4.5 per cent of the total forest land in the region (Xunta de Galicia, 2001). Although in some areas of Spain, Maritime pine and Scots pine stands have been successfully managed, only a very small proportion of the stands in Galicia have been managed by application of appropriate silvicultural regimes. Forest management requires tools that enable prediction of the development of forest stands under different silvicultural systems, to facilitate decision-making at both stand and forest levels. As management practices tend to intensify over time, the need for growth and mortality models becomes more apparent.

Crown attributes are key components of growth and yield models (Soares and Tomé, 2001), because crown shape and size influence production efficiency (Jack and Long, 1992), which is directly related to growth and mortality. Crown attributes are also important for assessing canopy cover (Gill *et al*., 2000), competitive level (Mitchell, 1975; Hann and Ritchie, 1988; Biging and Dobbertin, 1992), tree vigour (Ritchie and Hann, 1986; Hasenauer and Monserud, 1996), microclimate (Grace *et al*., 1987), wood quality (Maguire *et al*., 1991), biological diversity (Dubrasich *et al*., 1997), mechanical stability (Wilson and Oliver, 2000), behaviour under wind stress (Gardiner *et al*., 2000) and fire susceptibility (Keyes and O'Hara, 2002; Mitsopoulus and Dimitrakopoulus, 2007) and for characterizing leaf area distribution (Weiskittel *et al*., 2008), amongst other features. The following tree attributes are the most commonly measured/modelled: (1) height to the base of the live crown, which together with total height enables estimation of the crown length and crown ratio; (2) crown width, assessed as maximum potential (i.e. for an open-grown tree) or largest crown width; (3) height to the largest crown width, to determine the competitive level of the lower part of the crown and (4) crown radius, which is useful for developing crown profile equations, which in turn are useful for determining crown volume and cross sectional areas.

Indirect and direct approaches have been used to describe the entire tree crown shape or profile (Marshall *et al*., 2003). For example, deterministic models estimate variables that characterize the branches along the stem to indirectly estimate the crown width (Cluzeau *et al*., 1994; Deleuze *et al*., 1996). Fractal analysis (Zeide and Gresham, 1991) and architectural models, based on stochastic sensitive growth grammars (Kurth and Sloboda, 1997), have also been used to describe the tree crown structure indirectly. However, deterministic (Mitchell, 1975; Biging and Wensel, 1990; Pretzsch, 1992) and stochastic models (Biging and Gill, 1997) have been widely used for direct prediction of crown width from tree attributes because of their simplicity and accuracy.

The method most commonly used to model tree crowns directly is to divide them into upper (i.e. mostly sun needles) and lower (i.e. mostly shade needles) crowns, separated at the point of the largest crown radius. A variety of equations have been used to describe both profiles. Hatch *et al*. (1975), Mawson *et al*. (1976) and Pretzsch (1992) used simple geometric shapes, although these were thought to be excessively rigid, and researchers therefore began to use more flexible models to represent crown shapes. Mitchell (1975) and Ottorini (1991) used the distance from the top of the tree to each point as a predictor variable in their crown radius equations. Kändler (1986), Mohren (1987), Biging and Wensel (1990), Hann (1999) and Rautiainen and Stenberg (2005) used variable exponent models to obtain different crown shapes, by changing the value of the parameters for different species. Crecente-Campo *et al*. (2009a) used the variable exponent model of Hann (1999) for the upper crown but fitted it simultaneously with a lower crown model and a height to the largest crown radius model. Hann (1999), Marshall *et al*. (2003) and Crecente-Campo *et al*. (2009a) demonstrated the effects of including modelling values of largest crown radius in the crown profile estimations, but did not develop models including this variable in a simultaneous fitting process.

The objective of the present study was to develop crown profile models for use when a limited number of measurements are available. These models will be useful for developing individual tree growth models (by enabling calculation of competition indices), as well as crown fire risk models, as in Crecente-Campo *et al*. (2009b) and Ruiz-González and Álvarez-González (2011), in the study area. We simultaneously fitted the equations used for crown profile estimation, with the purpose of developing a model including an equation for the largest crown width; this also allowed us to take into account correlations between the different variables and to use a different number of observations for the different variables involved in the fitting process.

Methods

Study area and data description

Data on Maritime pine were obtained from permanent plots installed in pure even-aged stands of this species in Galicia. A network of 25 permanent plots was installed in 2005 to obtain data for developing an individual tree growth model, and the trees were remeasured in 2009. However, as a result of forest fires and clear cutting, the 25 plots were not all available in 2009. A network of 30 thinning sites was installed in 2009 in the autonomous communities of Galicia, Asturias and León, to analyse the effects of thinning on growth, mechanical stability and forest fire risk in these stands. A subset of 25 plots was selected in 2009 from both networks in Galicia, coinciding with measurement of crown variables for an inventory. The plots were subjectively selected to represent a broad range of sites, stand conditions and ages of the Maritime pine population in northwestern Spain.

Data on Scots pine were obtained from a network of permanent plots established in pure plantations in Galicia in 1996. Thirty plots were selected in 2009 for measurement of crown variables, also coinciding with an inventory. As with Maritime pine, the plots were subjectively selected to represent a wide variety of sites, stand conditions and ages from the Scots pine population in northwestern Spain.

All the selected plots were installed in unthinned stands or stands thinned lightly from below (A or B degree of severity according to Smith *et al*.,1997). Thinning was not carried out after plot installation. All selected plots can be considered as fully occupying the growing space when the crown data were taken (i.e. in 2009), as regular mortality was observed.

In each selected plot, eight trees with healthy crowns were selected across size classes (i.e. dominant or co-dominant, intermediate and suppressed) for crown profile measurement. In each tree, the crown radius (CR_j, m) was measured at several points (a minimum of six) along the profile; the diameter (d, cm) was measured at breast height (1.3 m above ground), to the nearest 0.1 cm, and total tree height (HT, m) was measured to the nearest 0.1 m. The height to the base of the live crown (HBLC, m), defined as the point on the stem of the lowest live branch above which there were at least two consecutive live branches, and the largest crown radius (LCR, m, two measures taken at right angles) were also measured to the nearest 0.1 m in each tree. Summary statistics for the fitting dataset and the plots are shown in Table 1.

To measure the crown profiles, a device called a crown window, similar to that described by Hussein *et al*. (2000), was used. This device, which is based on similar triangles, consists of a vertical clear plastic sheet (25 × 40 cm) fixed to the ground, parallel to the axis defined by the tree bole. A grid scale is superimposed on the plastic sheet as a reference. Crown measures are taken by moving away from the tree until the full crown is visible through the crown window; the outer points of the crown profile are then drawn on the grid scale, without moving the point-of-view or the crown window. The first point drawn was the base of the full live crown (i.e. the point at HBLC), and the last was the crown tip (i.e. total tree height) (Figure 1). The height to the base of the live foliage (HBLF, m) was defined as the height from ground to the first live needles of the branches taken as the HBLC. The other points were placed on the crown profile, as evenly as visual judgment allowed, to trace the main 'turn' in the crown profile curve (Rautiainen and Stenberg, 2005), so that more points were usually obtained on larger crown trees. Each crown was measured in a random direction by randomly selecting a compass direction to minimize possible direction effects. In some cases, the tree crown could not be viewed from the randomly selected direction and another direction was selected at random. Crown profiles were then obtained for both sides of each tree. The crown profile measures for each tree were considered independent since each crown radius measure was taken at a different height above crown base (CH_j, m, i.e. the vertical height from HBLF to each crown radius) for the right and the left profile.

Visual methods have shown acceptable accuracy for measuring crown radii and the corresponding height (Hussein *et al*. 2000; Rautiainen and Stenberg, 2005) and for measuring other crown characteristics (Martín-García *et al*., 2009). Davies (2006) compared visual methods and photogrammetric techniques to measure crown profiles and found no obvious difference in accuracy.

In the laboratory, the plastic sheets were scanned and digitized, and the digital images were then scaled using AutoCAD®. The coordinates of each point of the outer crown profile were obtained using ArcView® and the *adxycoo.ave* script. The ratio of the crown length to the base of the full live crown (calculated as the difference between HT and HBLC, divided by the crown length measured on the grid scale) was the scale factor used to calculate the CR_j and the CH_j at each measurement point.

Maguire and Hann (1989) found that crown foliage weight was most closely correlated with the surface area of the crown. Subsequently, several authors (Hann, 1999; Marshall *et al*., 2003; Crecente-Campo *et al*., 2009a) used HBLF as the definition of the crown base (Figure 1). We therefore also used HBLF in this study as the crown base (note that this also prevents the problem of HBLF being below HBLC, and thus having negative CH_j values).

Crown profile modelling

Once crown radius (CR_j) and height above crown base (CH_j) were obtained, we divided crowns into upper (i.e. mostly sun needles) and lower (i.e. mostly shade needles) crowns, separated at the height above ground at which

Table 1 Statistics of the fitting data set

Species	Variable	Number of observations	Mean	S.D.	Min.	Max.
Pinus Pinaster	*d* (cm)	200	29.16	11.61	7.20	67.10
	HT (m)	200	18.69	3.95	8.00	30.20
	MCR (m)	200	2.04	0.93	0.45	4.47
	CR (m)	2753	1.44	1.10	0	5.89
	CH (m)	2753	3.22	2.57	0	12.77
	LCR (m)	400	2.17	1.12	0.54	5.89
	CL (m)	400	5.59	2.36	1.23	12.77
	HBLF (m)	400	13.11	3.11	6.00	22.17
	HLCR (m)	400	14.75	3.46	6.77	25.82
	HT/*d*	200	0.71	0.20	0.33	1.57
	HBLF/HT	400	0.70	0.10	0.40	0.91
	N (trees ha^{-1})	25	794	355	270	1990
	G ($\text{m}^2\ \text{ha}^{-1}$)	25	20.80	5.50	11.77	37.36
	Hm (m)	25	17.53	3.18	13.40	26.04
Pinus sylvestris	d (cm)	240	22.25	7.63	5.95	46.1
	HT (m)	240	15.87	4.42	3.50	27.60
	MCR (m)	240	1.77	0.73	0.45	4.23
	CR (m)	3088	0.79	0.61	0	3.58
	CH (m)	3088	2.11	1.69	0	8.13
	LCR (m)	480	1.21	0.62	0.20	3.58
	CL (m)	480	3.69	1.52	0.77	8.13
	HBLF (m)	480	12.18	4.19	1.80	22.62
	HLCR (m)	480	13.27	4.29	2.65	24.65
	HT/*d*	240	0.75	0.20	0.31	1.60
	HBLF/HT	480	0.69	0.12	0.23	0.96
	N (trees ha^{-1})	30	1025	380	411	1824
	G ($\text{m}^2\ \text{ha}^{-1}$)	30	45.39	14.61	14.86	81.06
	Hm (m)	30	15.62	3.98	6.19	21.90

d, diameter at breast height over bark (1.3 m above ground level); HT, total tree height; MCR, maximum crown radius for an open-grown tree; CR, crown radius; CH, crown height; LCR, largest crown radius; CL, crown length; HBLF, height to the base of the live foliage; HLCR, height to the largest crown radius; N, stems per hectare; G, basal area per hectare; Hm, mean height.

largest crown radius occurs (HLCR). We then used a system of two equations to represent the crown profile:

$$\mathrm{CRU}_j = \mathrm{LCR}\left(\frac{\mathrm{CL}-\mathrm{CH}_j}{L_U}\right)^{a_0+a_1(\mathrm{CL}-\mathrm{CH}_j/L_U)^{1/2}+a_2(\mathrm{HT}/d)} \qquad (1)$$

$$\mathrm{CRL}_j = \mathrm{LCR}\left(b_1+(1-b_1)\left(\frac{\mathrm{CH}_j}{L_L}\right)^{b_2}\right) \qquad (2)$$

where CRU_j is the upper crown radius (m) at each measurement point j; CL is the crown length (m, i.e. HT-HBLF); L_U is the length of the upper crown (m, i.e. HT-HLCR, see Figure 1); CRL_j is the lower crown radius (m) at each measurement point j; L_L is the length of the lower crown (m, i.e. HLCR-HBLF, see Figure 1); a_i and b_i are parameters to be estimated; and the other variables are as previously defined.

Equation (1) represents a truncated symmetric curve. The function describing the exponent of Equation (1) allows for changes in crown radius within the tree crown profile and also between trees with different slenderness ratios (i.e. HT/d), as a measure of tree social status. Equation (1) has been successfully applied to several species (Hann, 1999; Marshall *et al.*, 2003; Crecente-Campo *et al.*, 2009a; Hann *et al.*, 2011).

Equation (2) has been found to accurately characterize the lower crown profiles of radiata pine (*Pinus radiata* D. Don) (Crecente-Campo *et al.*, 2009a) and red alder (*Alnus rubra* Bong.) (Hann *et al.*, 2011). The b_1 parameter defines the width of the crown at crown base, whereas parameter b_2 determines the shape of the lower crown profile.

In a preliminary analysis, this two-equation system performed better than other models and simple geometric forms (i.e. cones and ellipses) for estimating crown profiles, as has been found for radiata pine in Spain (Crecente-Campo *et al.*, 2008), and it was therefore selected for both species.

Modelling other crown variables

Values of LCR, CL, CH_j, L_U and L_L are required to use the fitted crown profile models. In addition, HT and d are required for Equation (1). Of these, HT and *d* are commonly measured variables, and CH_j is simply the distance from HLBF (i.e. the independent variable) into the crown. The use of electronic hypsometers has considerably reduced the time required to obtain additional crown measurements, and CL is now more commonly measured. However, LCR and the height above ground at which it occurs (HLCR, m), used to determine L_U and L_L, are not usually measured. Thus, to apply the fitted crown profile models, estimated values of LCR and HLCR may be required as input variables. Several models have been tested for this purpose.

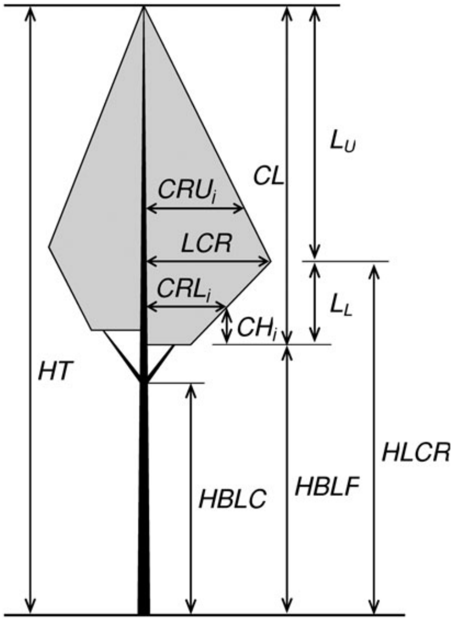

Figure 1 Variables used to characterize individual tree crowns. HBLC, height to the base of the live crown (m); HBLF, height to the base of the live foliage (m); HLCR, height to the largest crown radius (m); L_U, length of the upper crown (m); L_L, length of the lower crown (m); CRU_j, upper crown radius (m); CRL_j, lower crown radius (m); LCR, largest crown radius (m); CH_j, height from the base of the live foliage to each measurement point j (m); CL, crown length (m); HT, total tree height (m).

For modelling HLCR, we used a logistic function, restricted to predict values between HLBF and HT (Equation 3). We used the model formulation suggested by Hann (1997), and used in several other studies (Hann, 1999; Marshall *et al*., 2003; Hann *et al*. 2011), to model LCR (Equation 4):

$$\mathrm{HLCR} = \mathrm{HBLF} + \frac{\mathrm{HT} - \mathrm{HBLF}}{1 + \exp(X\alpha)} \tag{3}$$

$$\mathrm{LCR} = \mathrm{MCR} \cdot \mathrm{CRa}^{(X\alpha)} \tag{4}$$

where X is a vector of independent variables, α is a vector of parameters to be estimated, MCR (m) is the maximum crown radius resulting from open-grown trees and *CRa* (m) is the crown ratio (i.e. (HT-HBLF)/HT).

We estimated MCR by using the equations to estimate the maximum crown width for these species in Spain (Condés and Sterba, 2005). We divided the proposed equations by two to estimate of MCR for *P. pinaster* (Equation 5) and *P. sylvestris* (Equation 6):

$$\mathrm{MCR} = \exp\frac{(-0.9438 + 0.8371 \cdot \log(d))}{2} \tag{5}$$

$$\mathrm{MCR} = \exp\frac{(-0.2911 + 0.6716 \cdot \log(d))}{2} \tag{6}$$

To select predictor variables, we first linearized Equations (3) and (4) by rearranging the terms and using a logarithmic transformation, thus obtaining Equations (7) and (8), respectively:

$$\ln\left(\frac{\mathrm{HT} - \mathrm{HBLF}}{\mathrm{HLCR} - \mathrm{HBLF}} - 1\right) = X\alpha \tag{7}$$

$$\frac{\ln(\mathrm{LCR}/\mathrm{MCR})}{\ln(\mathrm{CRa})} = X\alpha \tag{8}$$

We fitted Equations (7) and (8) by using the REG SAS/STAT® procedure (SAS Institute Inc. 2009). We tested all possible combinations of predictor variables and selected the best combination by considering the square root of the mean squared error (RMSE), the significance of variables ($\alpha = 0.05$), measures of multicollinearity of predictor variables, specifically the condition number (Belsley, 1991) and biological reasoning, along with variables used in previous studies (Hann, 1997; Marshall *et al*., 2003). After selecting the best combination of variables, we fitted the models using nonlinear regression to look for consistency and significant parameter estimates. The following expressions provided the best results:

$$\mathrm{HLCR} = \mathrm{HBLF} + \frac{\mathrm{HT} - \mathrm{HBLF}}{1 + \exp(c_0 + c_1\mathrm{HT} + c_2(\mathrm{HT} - \mathrm{HBLF}) + c_3/d)} \tag{9}$$

$$\mathrm{LCR} = \frac{\exp(-0.9438 + 0.8371 \cdot \log(d))}{2} \cdot \left(\frac{\mathrm{HT} - \mathrm{HBLF}}{\mathrm{HT}}\right)^{(e_0 + e_1/d + e_2(\mathrm{HT} - \mathrm{HBLF}))} \tag{10}$$

$$\mathrm{LCR} = \frac{\exp(-0.2911 + 0.6716\log(d))}{2} \cdot \left(\frac{\mathrm{HT} - \mathrm{HBLF}}{\mathrm{HT}}\right)^{(e_0 + e_1 d + e_2 d/\mathrm{HT} + e_3(\mathrm{HT} - \mathrm{HBLF}))} \tag{11}$$

where c_i and e_i are the parameters to be estimated and the other variables are as previously defined. Equation (9) adequately characterized the HLCR for both species. Equation (10) adequately characterized LCR for *P. pinaster*, and Equation (11) was adequate for *P. sylvestris*.

Although crown attributes can be affected by stand density (Ritchie and Hann, 1987; Larocque and Marshall, 1994), from a practical point of view, inclusion of density variables results in an instant change in predicted crown attributes following thinning, which is not realistic. Thus, following the results of some authors (Hann, 1997; Pretzsch *et al*., 2002; Crecente-Campo *et al*., 2009a), we only included tree variables in the previous equations.

Model fitting

At this point, we defined two different crown models. We selected crown profile model 1 (CPM1), which comprised Equations (1), (2) and (9), as the basic model, because HLCR is a variable that is not likely to be measured. LCR and HBLF were considered to be measured in CPM1. Such a model should be used when measured values of d, HT, LCR and HBLF are available.

Crown profile model 2 (CPM2) comprised Equations (1), (2), (9) and (10) for Maritime pine, and Equations (1), (2), (9) and (11) for Scots pine. This model should be used when measured values of d, HT and HBLF are available.

We fitted the system of equations that form each defined crown profile model simultaneously to account for likely cross-equation correlations of error terms, as described further later. We used the MODEL procedure of SAS/ETS® (SAS Institute Inc., 2008), which includes full information maximum likelihood (FIML), to fit each system of equations. We obtained the initial parameter values by fitting each equation in the system separately, using the NLIN procedure of SAS/STAT® (SAS Institute Inc., 2009). We used different initial values used to ensure that a global minimum was achieved. As the HLCR and LCR values are the same for each profile, and because simultaneous fitting requires the same number of observations for all the variables, a special structure of the dataset was required. For each CR value, we included HLCR and LCR observations, so that there were m_i equal observations for crown profile i. In the fitting process, we then weighted the HLCR and LCR equations with the inverse of the number of observations in each profile (i.e. $1/\sqrt{m_i}$) (Diéguez-Aranda *et al*., 2006; Álvarez-González *et al*., 2007; Crecente-Campo *et al*., 2010), using the *resid.variable* option of SAS/ETS® MODEL procedure (SAS Institute Inc., 2008), which enables individual weighting of each equation (note that as this option acts on the residuals before they are squared, the root of the weight must be specified).

We initially fitted crown profile models 1 and 2 by assuming that the within-equation errors were independent and identically distributed (iid). However, since crown profile models are similar to stem taper models (in that several measures are used for each crown profile), autocorrelation amongst measures within a profile is likely (Crecente-Campo *et al*., 2009a). The variances of within-equation errors between profiles may also be heterogeneous. For single linear statistical models, the least squares estimates of regression coefficients remain unbiased and consistent in the presence of autocorrelation and heteroscedasticity, but they are no longer efficient (Myers, 1990). However, the usual estimates of standard errors of the coefficients are biased, invalidating statistical tests that use *t* or *F* distributions and confidence intervals (Kutner *et al*., 2005). The same is also true for systems of equations, as discussed by LeMay (1990), particularly when the models have the same regression variables, and when the dependent variable of a model acts as an independent variable in another model. To improve the efficiency of estimated parameters, we modelled the within-equation error structure by specifying the heteroscedastic, autocorrelated error structure and we fitted all of the equations that were part of each crown profile model simultaneously.

To account for within-profile autocorrelation, an autoregressive error structure, which enables the model to be applied to irregularly spaced unbalanced data (Diéguez-Aranda *et al*., 2005; Crecente-Campo *et al*., 2009a), was added to each CR equation for CPM1 and CPM2. This error structure expands the error term as follows:

$$e_{ij} = \sum I_m \rho_k^{h_{ij}-h_{ij-k}} e_{ij-k} + u_{ij} \tag{12}$$

where e_{ij} is the ordinary residual of the *j*th measure within the *i*th crown profile, e_{ij-k} is the ordinary residual of the *i*th profile and the $(j-k)$th measure, $I_k = 1$ when $j > k$ and 0 when $j \leq k$, ρ_k is the *k*-order autoregressive parameter, $h_{ij}-h_{ij-k}$ is the distance separating the *j*th from the $(j-k)$th observation within each profile *i*, with $h_{ij} > h_{ij-k}$, and u_{ij} is an independent, normally distributed error term, with a mean value of zero.

After autocorrelation was corrected, and to account for between-profile heteroscedasticity, the error variance for the error term (σ^2) was needed to calculate the weights to be applied to each CR observation as part of the fitting process. Although several models concerning the nature of heteroscedasticity have been identified, the error variances can often be modelled as a power function of an independent variable for tree-level models (Hann, 1999; Marshall *et al*., 2003; Crecente-Campo *et al*., 2009a). For the CR models, the error variance was modelled with a power function of LCR: $\hat{\sigma}_i^2 i \propto \mathrm{LCR}^q$. The method suggested by Park (1966) was used to regress squared residuals against LCR, to obtain an estimate of *q*, as follows: $\hat{u}_{ij}^2 = \gamma \times \mathrm{LCR}_i^q$. This expression was linearized by taking the natural logarithm:

$$\ln \hat{u}_{ij}^2 = \ln \gamma + q \times \ln \mathrm{LCR}_i \tag{13}$$

We then estimated the parameters of Equation (13) by using linear least squares regression. We subsequently included the *q* value in the weighting factor (*w*) for each crown profile equation as follows: $w = 1/\sqrt{\mathrm{LCR}_i^q}$ (the root of the weight was again specified because the *resid.variable* option acts on the residuals before they are squared).

After removal of autocorrelation, we used the residuals from Equation (12) to calculate weights (Equation (13)). We also weighted the HLCR and LCR equations by using HT and *d*, respectively (as the base of the power weighting function), residuals from the previous fitting as dependent variables and the previously explained methodology. After establishing the *q* parameter for each equation (using PROC REG of the SAS/STAT® system), we finally estimated all parameters by using FIML of the PROC MODEL. We tested he hypothesis that $\rho_k = 0$ for each *k* by using the fit results, and we used the test results to select the autoregressive order for each CR equation of CPM1 and CPM2. We include an example of the SAS code used to fit CPM2 in Appendix A.

Model performance

We used numerical and graphical analyses to compare CPM1 and CPM2. We also considered the following statistical criteria: the RMSE, as a measure of the accuracy of the estimates, and the pseudo R^2, which similar to the coefficient of determination for linear models that indicates the proportion of the total variance explained by the model. These were calculated as follows:

$$\text{Pseudo}\, R^2 = 1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{m_i}(y_{ij}-\hat{y}_{ij})^2}{\sum_{i=1}^{n}\sum_{j=1}^{m_i}(y_{ij}-\bar{y})^2} \tag{14}$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m_i}(y_{ij}-\hat{y}_{ij})^2}{(\sum_{i=1}^{n} m_i) - p}} \tag{15}$$

where y_{ij}, $\hat{y}_{ij}$ and $\bar{y}$ are the measured, estimated and average values of the dependent variable, respectively, *n* is the total number of profiles used, m_i is the number of observations in each profile and *p* is the number of model parameters. We calculated these goodness-of-fit statistics for the entire crown, for the upper and lower crowns separately, and for the other crown variables. We also constructed plots of predicted against observed values, and of residuals against predicted values, as helpful tools for identifying lack of fit.

Results

Parameter estimates for CPM1 and CPM2 are given in Table 2. Some parameters were not significantly different from zero ($\alpha = 0.05$) and were therefore removed from the models along with the associated independent variables. This parameter estimates had no practical impact on predictions from the equations, and their elimination did not change the biological behaviour of the models. The other parameters were highly significant ($\alpha = 0.01$) (Table 2). In particular, within-profile autocorrelation was significant for all the crown equations and only for $k = 1$. We set parameter b_2 of Equation (2) to 1, to obtain convergence for Maritime pine in CPM2.

Plots of weighted residuals against predicted values for CPM1 (Figure 2) showed well-distributed variance errors, indicating that the heteroscedasticity models used to estimate weights were effective. The results for CPM2 were similar.

Both crown profile models were quite accurate, as indicated by fit statistics (Table 3). For the entire crown, CPM1 accounted for ~87 and 94 per cent of the variance in crown radii for Maritime pine and Scots pine, with RMSEs of 0.40 and 0.15 m, respectively. As expected, CPM2 was less accurate than CPM1, with RMSEs increasing by ~155 and 168 per cent for Maritime pine and Scots pine, respectively.

For HLCR, the results CPM1 and CPM2 were very similar, whereas the Pseudo R^2 was lower and RMSE was higher for Maritime pine than for Scots pine. For estimation of LCR in CPM2, the Pseudo R^2 was again lower and the RMSE was higher for Maritime pine than for Scots pine.

Discussion

Different crown profile models have been proposed. Most of these are based on simple geometric forms (i.e. cones and ellipses), with simplicity promoted over complexity. However, preliminary analysis in this study showed that simple geometric forms were not accurate enough to represent the crown profiles for the species used, with lack of fit and biased estimations observed for both the upper

Table 2 Parameter estimates for CPM1 and CPM2, for Maritime pine (*P. pinaster*) and Scots pine (*P. sylvestris*)

Model	Species	Equation number	Parameter estimates		Approx. standard error
CPM1	*P. pinaster*	(1)	a_0	0.3695	0.0250
		(1)	a_1	0.2012	0.0478
		(2)	b_1	0.8218	0.0063
		(2)	b_2	7.184	1.682
		(9)	c_0	1.174	0.095
		(9)	c_2	−0.07323	0.00821
		(9)	c_3	5.029	0.878
	P. sylvestris	(1)	a_0	0.7962	0.0261
		(1)	a_1	−0.2848	0.0315
		(1)	a_2	−0.2424	0.0260
		(2)	b_1	0.7550	0.0048
		(2)	b_2	0.5169	0.0518
		(9)	c_0	0.3699	0.0654
		(9)	c_2	0.05059	0.00880
		(9)	c_3	5.905	0.713
CPM2	*P. pinaster*	(1)	a_0	0.2147	0.0166
		(1)	a_1	0.6386	0.0355
		(2)	b_1	0.7443	0.0076
		(2)	b_2	1[a]	–
		(9)	c_0	1.297	0.111
		(9)	c_2	−0.08765	0.00948
		(9)	c_3	3.972	1.508
		(10)	e_0	0.5156	0.0191
		(10)	e_1	−1.712	0.237
		(10)	e_3	−0.01789	0.00211
	P. sylvestris	(1)	a_0	0.6414	0.0291
		(1)	a_2	−0.1551	0.0389
		(2)	b_1	0.7134	0.0060
		(2)	b_2	0.4561	0.0526
		(9)	c_1	0.01508	0.00216
		(9)	c_2	0.05044	0.00707
		(9)	c_3	9.103	0.445
		(11)	e_0	0.7636	0.0116
		(11)	e_1	−0.01564	0.00052
		(11)	e_2	0.1972	0.0099
		(11)	e_3	−0.006250	0.002480

[a]This parameter was set at 1 to obtain convergence.

and lower crowns. The same was found for radiata pine in northwestern Spain (Crecente-Campo *et al*., 2008). Crecente-Campo *et al*. (2009a) analysed more flexible crown models and obtained the best results with the same functions as in the present study, which have also been used by Hann (1999), Marshall *et al*. (2003) and Hann *et al*. (2011).

We simultaneously fitted Equations (1), (2) and (9), for the upper crown radius, the lower crown radius and the height to the largest crown radius, respectively, to account for probable cross-equation correlations of error terms, in CPM1. We did the same by including Equations (10) and (11) in CPM2. To improve the efficiency of parameter estimation, we modelled the within-equation error structure by specifying a heteroscedastic, autocorrelated error structure. The only purpose of this specification is to work under iid error conditions, to prevent underestimation of the covariance matrix of the parameters, thereby making it possible to carry out the usual statistical tests (West *et al*., 1984). Model estimations were not very different from those obtained with models fitted without considering such correction. Therefore, autocorrelation and weighting parameters are disregarded in practical applications.

The term HT/d in Equation (1) was only significant for Scots pine. Marshall *et al*. (2003) also found that this term was not significant for western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) in Oregon. However, trees with high slenderness ratios are tall, thin trees with crowns that are expected to differ in shape from trees with lower slenderness ratios (i.e. short, wide trees). Hann (1999) found that, at the same relative point in the crown, dominant Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) trees in Oregon were more conical (i.e. larger exponents) than understory trees. The same has been found for radiata pine in northwestern Spain (Crecente-Campo *et al*., 2009a). Nevertheless, CPM1 and CPM2 include HT and *d* in the formulation of Equations (9) and (10), which we simultaneously fitted with Equations (1) and (2), and thus, the effects of HT and *d* in crown radius were finally also included for Maritime pine.

Equations (9) to (11) adequately characterized the HLCR and the LCR for both species (Table 3). These equations included restrictions to force LCR to be positive and to be equal to MCR when HBLF equals zero, and to limit HLCR estimates to between HBLF and HT. Although carefully formulated empirical equations may be more accurate than theoretical equations for a wide range of data, theoretically based equations may be more reliable for predictions that involve extrapolations beyond the range of the data (Vanclay, 1994). Some authors (Hann, 1997; Pretzsch *et al*., 2002; Crecente-Campo *et al*., 2009a) have used equations that constrain estimates to within logical bounds, and R^2 values from 0.48 to 0.88 were obtained, depending upon the species. The results of those studies are similar to those of the present study, in terms of explained variance and selected predictor variables.

The fitting statistics for the models developed in this study provided quite accurate results. The RMSE values for Scots pine were lower than those obtained for Maritime pine (Table 3). However, the results are not directly comparable, because the mean LCR values for the two species are quite different (2.17 m for Maritime pine and 1.21 m for Scots pine, see Table 1), and therefore, the magnitude of the error is different. To compare these results with those of other studies, we scaled the RMSE values by multiplying them by the ratio between the previously determined average LCR for a similar dataset (Crecente-Campo *et al*., 2009a) (1.60 m) and the average LCRs of the present dataset. This produced scaled RMSEs of 0.2855 and 0.1919 m for Maritime pine and Scots pine upper crown, respectively, for CPM1. These values are similar to those obtained for western hemlock in Oregon (Marshall *et al*., 2003) (0.2532 m) and radiata pine in northwestern Spain (Crecente-Campo *et al*., 2009a) (0.2296 m) and higher than those obtained for Douglas-fir in Oregon (Hann, 1999) (0.1474 m), in which the same model was used (all the values were also scaled to be comparable to ours). Concerning the lower crown, scaled RMSE values of 0.3215 and 0.1929 m for Maritime pine and Scots pine, respectively, were obtained for CPM1. These values were similar to those obtained for radiata pine in northwestern Spain (Crecente-Campo *et al*., 2009a) (0.2473 m) using the same model.

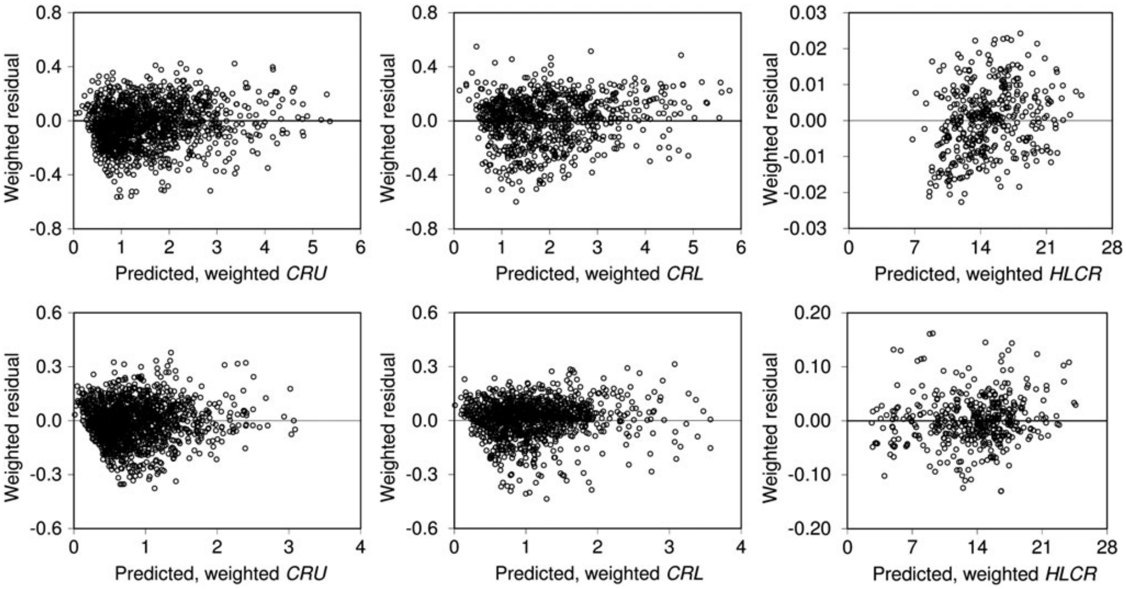

Figure 2 Weighted residuals plotted against predicted values for CPM1 for Maritime pine (first line) and Scots pine (second line).

Table 3 Goodness-of-fit statistics for CPM1 and CPM2, for Maritime pine (*P. pinaster*) and Scots pine (*P. sylvestris*)

Model	Species	Statistic	Entire crown	CRU	CRL	HLCR	LCR
CPM1	*P. pinaster*	Pseudo R^2	0.8668	0.8796	0.8214	0.8624	–
		RMSE	0.4015	0.3872	0.4360	1.288	–
	P. sylvestris	Pseudo R^2	0.9400	0.9452	0.9362	0.9829	–
		RMSE	0.1502	0.1415	0.1459	0.5634	–
CPM2	*P. pinaster*	Pseudo R^2	0.6791	0.7130	0.4228	0.8616	0.6633
		RMSE	0.6234	0.5982	0.7429	1.292	0.652
	P. sylvestris	Pseudo R^2	0.8311	0.8581	0.7228	0.9826	0.7661
		RMSE	0.2523	0.2337	0.3047	0.5671	0.2997

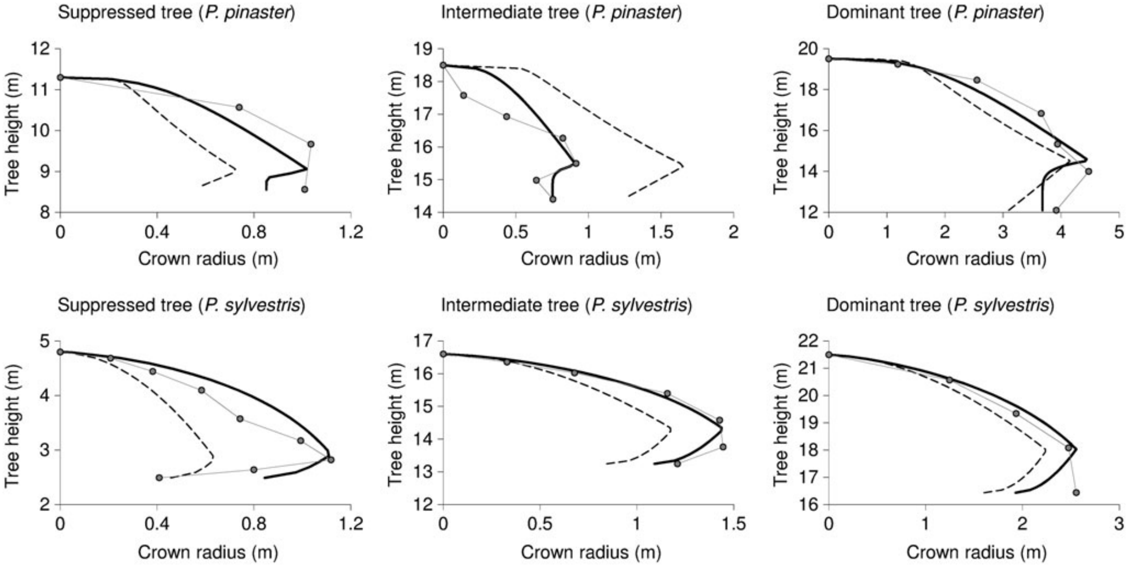

Figure 3 Appearance of the estimated crown profiles for CPM1 (full line) and CPM2 (dashed line) superimposed on real data (dots) for a suppressed, an intermediate and a dominant tree, for Maritime pine (*Pinus pinaster*) and Scots pine (*Pinus sylvestris*).

The estimated parameters for the models developed here make biological sense, although they should be interpreted with caution because the predictor variables are not completely independent. The model used to estimate LCR predicts a larger radius as tree diameter becomes larger for both species (Figures 3 and 4), and this value decreases for taller trees for constant crown lengths (Figure 4d and h) (i.e. for the same *d* and CL, a taller tree will have a smaller LCR). However, predicted LCR equals MCR when HBLF equals 0 for CPM2, and this value decreases as HBLF increases (Figure 4c and g). In Figure 4, we intentionally included large diameter and height values (outside the range of data) to show the biological realism of the models developed.

The models did not include stand variables in their formulation. Some authors have reported that crown attributes are affected by stand density (Ritchie and Hann, 1987; Larocque and Marshall, 1994) and age (Ishii and McDowell, 2002). However, trees adapt to the environment to improve mechanical stability and access to light, and the particular characteristics of each tree, as measured by the variables *d*, HT and CL, are the result of its response to its environment, including inter-tree competition. Therefore, the use of these variables may remove the need for density variables or competition indices (Crecente-Campo *et al*., 2009a). We attempted to include some stand variables in the LCR model for CPM2, but none of the tested variables (stems per hectare, basal area, dominant height, dominant diameter, mean height, mean diameter or quadratic mean diameter) yielded improvement in model performance.

CPM1 produced quite accurate results for both species, for estimating both crown radius and i HLCR. The major limitation of this model is that it requires inclusion of LCR and HBLF measurements.

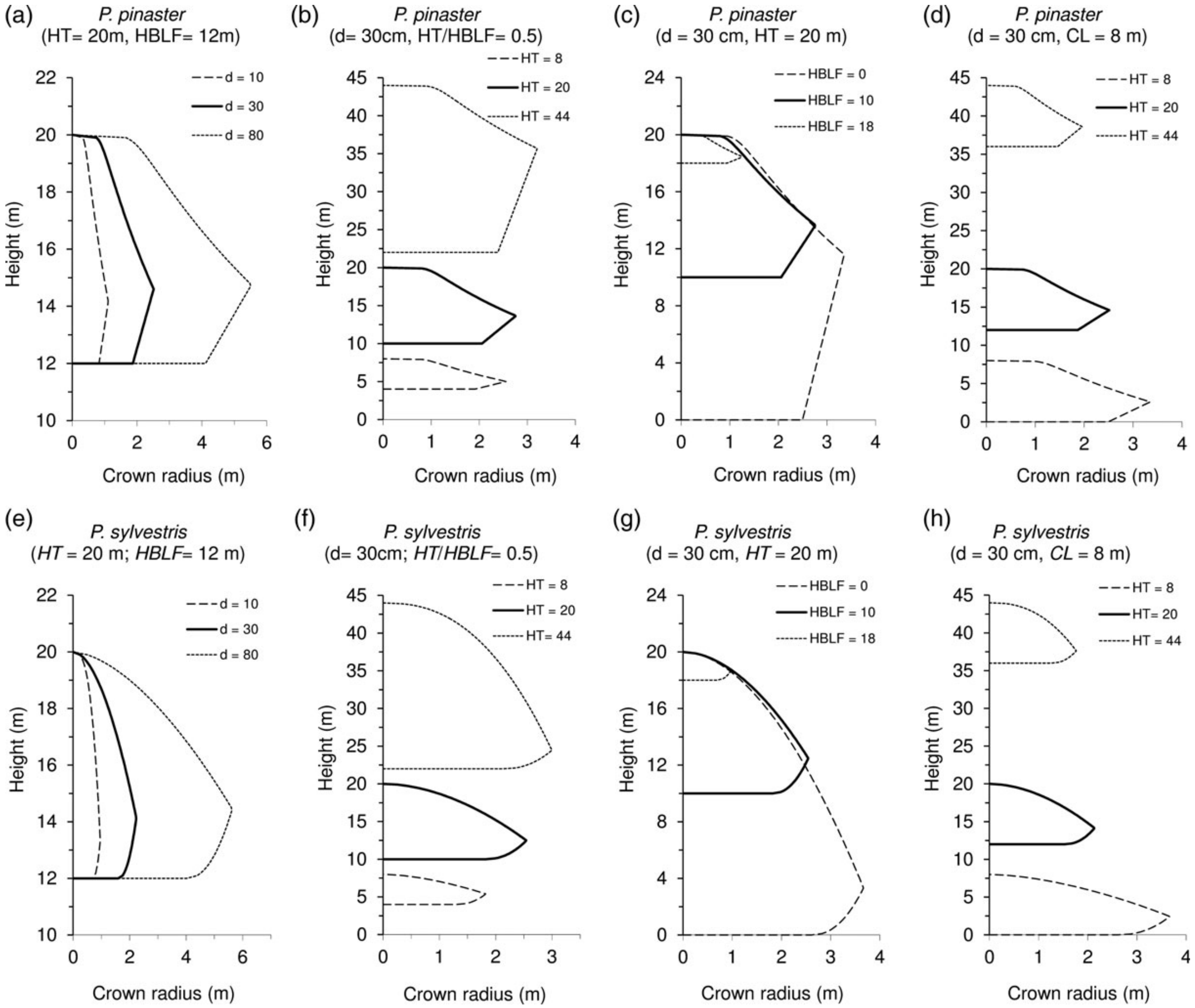

Figure 4 Example of the influence of diameter, height and height to the base of the live foliage in the crown profiles estimated by CPM2, for Maritime pine (*P. pinaster*) and Scots pine (*P. sylvestris*). Very large diameter and height values (outside the range of data) were intentionally used to show the biological realism of the developed models.

However, if LCR is not measured, crown radius estimates, HLCR and LCR can be easily obtained using CPM2, although the accuracy of the results is clearly reduced (Table 3), independently of the tree status.

We also attempted to model HBLF and include this equation as part of the system of equations used to estimate crown radius. However, this proved impossible, for several reasons. The main problem was related to the change in the crown base. All points in the crown below the predicted height to the base of the live foliage ($\widehat{HBLF}$) would have a predicted a crown radius of zero. This restriction must be included in model fitting (i.e. the predicted radius for the points below $\widehat{HBLF}$ is 0). This restriction led to convergence-related problems. Without using this restriction, convergence was achieved using a simple HBLF logistic model, bounded to predict values between 0 and HT using only tree-level variables, but the crown profile model predictions were not biologically realistic. Moreover, although some authors use only tree-level attributes in their equations for HBLF, several authors have demonstrated that stand-level attributes, such as indicators of one-sided competition (e.g. basal area in larger trees or crown competition factor in larger trees), two-sided competition (e.g. basal area or crown competition) and site productivity (e.g. site index) are important predictors of HBLF (Ritchie and Hann, 1987; Hasenauer and Monserud, 1996; Soares and Tomé, 2001; Hann *et al*., 2011). Convergence was not achieved by inclusion of stand-level variables in the HBLF equation. We therefore chose to require measurement of convergence (as was done by Hann (1999), Marshall *et al*. (2003) and Crecente-Campo *et al*., 2009a) for use in CPM1 and CPM2.

Although specific types of forest models are generally required for trees growing in different locations, site fertility and/or structural stand types (Saunders and Wagner, 2008; Crecente-Campo *et al*., 2009c), the structure of the models developed here may be used in other areas of Spain by using local data to estimate the parameters, since they only include tree-level variables in their formulation, and the models for estimating MCR (Condés and Sterba, 2005) were developed for the entire country.

Crown cross sectional areas at different heights, crown volume at different heights and total crown volume are the components of many competition indices found to be superior to other competition indices that do not include crown dimensions in their formulation (Biging and Dobbertin, 1992; Biging and Dobbertin, 1995; Schröder et al., 2007). Although these indices are of particular interest in mixed forests, which have complex structures with several strata (Schröder *et al*., 2007), they show good potential for use in forest plantations. The models developed in this study enable calculation of crown volume and crown cross sectional areas by integration of the crown radius equation, thus facilitating calculation of such competition indices.

The use of this type of model also enables development of species-specific equations for estimating canopy variables related to the potential of crown fires in coniferous stands, such as canopy base height and the canopy bulk density. These canopy variables are relatively difficult to quantify and also difficult to define accurately; however, crown profile models have proven to be very effective tools for this purpose (Ruiz-González and Álvarez-González, 2011).

Further studies relating crown volume and surface area (variables that easily obtained with the models developed in this study) to dry crown biomass, carbon content and leaf area index (LAI) would provide important tools for assessing carbon storage in the forests in the region.

Conclusion

A system of equations including a model for the upper crown (above the largest crown radius), a model for the lower crown (below the largest crown radius) and a model for the height to the largest crown radius (which separates the upper from the lower crown) provided accurate estimates of the crown shape for two conifer species in northwestern Spain. This system requires measurements of diameter, height, height to the base of the live foliage and largest crown radius as input variables. If the largest crown radius is not available, an alternative system of equations, which also includes a model for the largest crown radius, can be used.

The components of both systems of equations were fitted simultaneously to take into account correlations between the different variables. The use of independent weighting factors for each of the models enabled consideration of an autocorrelated, heteroscedastic error structure and use of a different number of observations for the different variables involved in the fitting process.

Conflict of interest statement

No conflicts of interest.

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Appendix A

Example of the SAS code used to fit the simultaneous system CPM2 for Maritime pine. Lines [1] to [15] are used to create new variables to include a 3-order autocorrelation structure in the model, according to Equation (12). Lines [20] and [22] define the expression of Equations (9) and (10), respectively. Lines [24] to [37] define the expression of Equations (1) and (2). An 'if' statement is used to analyse the position of the measured crown length (CRi) with regard to the maximum crown length (LCR). Lines [26] to [28] and lines [33] to [35] are used to include the 3-order autoregressive structure for Equations (1) and (2), respectively. Lines [21], [23], [29] and [36] define the weighting factors used to model the error variance in Equations (9), (10), (1) and (2), respectively. Parameters ki $\{i = 1, \dots 4\}$ were previously obtained using Equation (13). Lines [21] and [23] also include the number of observations in each profile (mi) in the weighting factor because of the special structure of the dataset.

```
[1] data fit_database;
[2] set original_database;
[3] dist1 = CHi-lag(CHi); #estimate the distance between measures
[4] if PROFILE ne lag(PROFILE) then dist1 = 0; # distances = 0 for different profiles
[5] if dist1 ne 0 then I1 = 1;
[6] else I1 = 0;
[7] dist2 = CHi-lag2(CHi);
[8] if PROFILE ne lag2(PROFILE) then dist2 = 0;
[9] if dist2 ne 0 then I2 = 1;
[10] else I2 = 0;
[11] dist3 = CHi-lag3(CHi);
[12] if PROFILE ne lag3(PROFILE) then dist3 = 0;
[13] if dist3 ne 0 then I3 = 1;
[14] else I3 = 0;
[15] run;
[16] proc sort data = fit_database; by TREE PROFILE CHi; run;
[17] proc model data = fit_database;
[18] parms a0 a1 a2 b1 b2 c0 c1 c2 c3 e0 e1 e2 rho1 rho2 rho3 gam1 gam2 gam3;
[19] exogenous HBLF HT D;
[20] HLCR = HBLF + (HT−HBLF)/(1 + exp(c0 + c1*HT + c2*(HT−HBLF) + c3/D));
[21] resid.HLCR = resid.HLCR/sqrt(mi*HT**k3);
[22] LCR = 1/2*exp(−0.9438 + 0.8371*log(D))*((HT−HBLF)/HT)**(e0 + e1/d + e2*(HT−HBLF));
[23] resid.LCR = resid.LCR/sqrt(mi*D**k4);
[24] if CHi > (HLCR-HBLF) then do;
[25]    CRi = LCR*((CL−CHi)/(CL−(HLCR−HBLF)))**(a0 + a1*((CL−CHi)/(CL−(HLCR−HBLF)))**.5 + a2*HT/D)
[26] -I1*rho1**dist1*zlag1(resid.CRi)
[27] -I2*rho2**dist2*zlag2(resid.CRi)
[28] -I3*rho3**dist3*zlag3(resid.CRi);
[29] resid.CRi = resid.CRi/sqrt(LCR**k1);
[30] end;
[31] else do;
[32] CRi = LCR*(b1 + (1-b1)*(CHi/(HLCR−HBLF))**(b2))
[33] -I1*gam1**dist1*zlag1(resid.CRi)
[34] -I2*gam2**dist2*zlag2(resid.CRi)
[35] -I3*gam3**dist3*zlag3(resid.CRi);
[36] resid.CRi = resid.CRi/sqrt(LCR**k2);
[37] end;
[38] fit CRi HLCR LCR start = (# initial values of the parameters #) /FIML;
[39] run;
```